Implications of constant growth of abnormal earnings in perpetuity for equity premia, discount rates, earnings, dividends, book values and key financial ratios. An extension of Claus and Thomas. *

by

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Abstract

We derive analytical formulas for the post-horizontal and asymptotic behavior of earnings, dividends, book value, and key financial ratios, as implied by the terminal value model of constant perpetual abnormal earnings growth. The implications of Claus and Thomas (2001) (CT) abnormal earnings growth forecasts for these quantities are examined and found reasonable. Analysis of the implicit functional relationships between the equity premium and the aforesaid quantities using CT's U.S. data reveals that a traditional premium of 8% implies 14% asymptotic growth in abnormal earnings, earnings, dividends and book value, and equally extreme asymptotic return-on-equity, price-to-earnings and price-to-book ratios.

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Implications of constant growth of abnormal earnings in perpetuity for equity premia, discount rates, earnings, dividends, book values and key financial ratios. An extension of Claus and Thomas.

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In the October 2001 issue of Journal of Finance, Claus and Thomas (CT hereafter) perform an interesting analysis of the equity risk premium using a novel approach. The equity risk premium, the difference between the expected return on the stock market and the risk-free rate, is a central quantity in finance. The most commonly used estimate of the U.S. risk premium is probably the one provided by Ibbotson Associates in their annual publication Stocks, Bonds, Bills and Inflation. This estimate of the equity risk premium is based on the historical performance of various portfolios of U.S. stocks and bonds since 1926, and CT report that the estimate has been between 7% and 9% for recent years. CT note that such large premia exceed "estimates derived from theory, from other periods and markets, and from surveys of institutional investors." (p. 1629) By using the abnormal earnings model (AEM), an accounting based valuation model, in conjunction with analysts' earnings forecasts, the authors are however able to obtain lower and perhaps more reasonable empirical estimates of the equity risk premium.

According to the abnormal earnings model, which is equivalent to the dividend discount model, present value equals current book value of equity plus future abnormal earnings discounted at the cost of equity capital. Basically, the authors input current book value of equity, five years' of explicit analysts' earnings forecasts. For the period beyond year +5, for which there are no explicit analysts forecasts, they compute a terminal value (or continuing value) assuming that abnormal earnings of year +5, the last year with explicit forecasts, grow at a constant rate (equal to forecasted inflation) in perpetuity. They then solve for the discount rate that equates observed market value to present value according to the abnormal earnings model. From this discount rate, they subtract the risk-free rate to get the risk premium. The so obtained equity risk premia lie around 3% for the post-1985 period and are thus considerably lower - about 5% lower - than the Ibbotson estimate.

This paper extends CT's analysis in several respects. We show that in the standard framework of AEM analysis (assuming a constant dividend payout ratio), the seemingly simple terminal value model of constant perpetual growth in abnormal earnings beyond the explicit forecast horizon implies specific, non-trivial post-horizon behavior of future earnings, book values and dividends. We derive explicit formulas for the post-horizon behavior of earnings, book values and dividends. Among other things, these formulas permit us to show that the asymptotic growth rates of earnings, dividends and book values will converge to either the retention rate (the discount rate times one minus the payout ratio) or the perpetual growth rate of abnormal earnings, depending on what term is the greater one. CT argue that the AEM offers an advantage over the Gordon dividend growth model (GDM) in that "future streams for a number of value-relevant indicators, such as price-to-book ratios..., price-to-earnings ratios..., and accounting return on equity..., can also be projected under the abnormal earnings approach. This allows one to paint a more complete picture of the future for different assumed growth rates." (p. 1637).[1] However, without explicit formulas for the behavior of earnings, dividends and book values, projection of the "value-relevant indicators" requires iterative procedures. In response to this, we derive explicit formulas for the post-horizontal and asymptotic behavior of the price-to-earnings ratio, the price-to-book ratio and return-on-

[1] This statement is difficult to understand, because the GDM and a constant dividend payout ratio assumption permit the projection of future streams of the same variables; in fact, CT do this themselves in Figure 2. It may even be simpler, since dividends and earnings grow at a constant rate.

equity. CT (p. 1664) write that other studies suggest that, under so-called conservative accounting, return-on-equity approaches the discount rate, but remains above it in the long-term. In the framework employed here, (perhaps as opposed to conservative accounting then), this is however only true when the retention rate is larger than the perpetual growth of abnormal earnings. Otherwise return-on-equity converges to the abnormal earnings growth rate divided by one minus the payout ratio. In addition, we derive the converging price-to-earnings and price-to-book ratios, which each also take either of two forms depending on the relative magnitudes of the retention rate and the abnormal earnings growth rate.

The derived formulas enable diagnostic checking of the implications of CT's empirical modeling. For instance, CT assume that abnormal earnings will grow in perpetuity at the rate of expected inflation, which they forecast as the 10-year risk-free rate minus 3%. With the explicit formulas at hand, it is possible to exactly analyze what this forecast implies about the growth of earnings, dividends and book values beyond the explicit forecast horizon. The expected inflation for 1991 is forecasted to 5.04%. Given the forecast abnormal earnings at the end of the forecast horizon, the discount rate, a dividend payout ratio of 50% of earnings, and the assumption that abnormal earnings grow at a rate of 5.04% in perpetuity, earnings and dividends will grow at a rate that initially is 7.33% and then slowly converges to 5.53%. For CT's U.S. sample, there are new forecasts stated in April each year between 1985 and 1998. For each such forecast, we calculate - which is straightforward with the derived formulas at hand - the implied future streams of future earnings, dividends, book values, and return-on-equity. The implied streams agree quite well with historical evidence, and for earnings and dividends, the average asymptotic growth rate equals 5.6%.

For a given year, the analysis just discussed concerned the implications of a single forecasted post-horizontal abnormal earnings growth rate. We extend that analysis by considering the equity premium and the discount rate as implicit functions of the forecast growth rate in abnormal earnings. This approach together with the algebra of the AEM valuation model, provide upper and lower bounds for the value of the equity risk premium. For the period studied, 1985-1998, the analysis suggests that traditional equity premium estimates of say 8-9% are very high and in some cases even algebraically impossible; for instance, given a payout ratio of 0.5 and a risk-free rate of around 8%, an equity premium of around 8% corresponds to an average growth rate of abnormal earnings in perpetuity of around 14%, which, in turn, imply converging growth rates of earnings, dividends and book values of the same magnitude. Another notable finding is that the asymptotic price-to-earnings ratio as a function of abnormal earnings growth g in general is not monotonically increasing, that is, it actually decreases as g (and r) increases for a part of the permissible domain.

CT also estimate the equity risk premium with the GDM and find it approximately equal to the Ibbotson estimate. It may appear disturbing that the premia obtained with the GDM differ substantially from the ones obtained with the AEM, since the GDM is a special case of the DDM, and the DDM and the AEM are “isomorphic” or equivalent. Applied the way CT apply them, and CT are careful to point this out, the GDM is not equivalent to the AEM. The different premium estimates should thus not come as a surprise. The DDM and the AEM are however equivalent (when applied consistently), so, for completeness, we show analytically that if the DDM is fed with the infinite dividends stream implied by the AEM, premium estimates coincide.

In section I, we briefly present the abnormal earnings model and derive the explicit formulas for post-horizontal behavior of earnings, book values and dividends implied by the terminal value assumption of constant growth of abnormal earnings in perpetuity. Based on these formulas, equations for the asymptotic behavior of earnings, dividends, book values, return-on-equity, price-to-book and price-to-earnings ratios are derived. It is also shown that estimates of the risk premium from the DDM and the AEM coincide when the first model is applied to the dividends stream implied by the latter model. Section II contains the results from the application of these explicit formulas to the U.S. data analyzed by CT. In particular, for every year between 1985 and 1998, we examine the future behavior of earnings, dividends, book values and key financial ratios given the forecasts of abnormal earnings growth employed by CT. We also check how that future behavior compares to available historical evidence. The extended analysis that considers equity premia, discount rates, earnings, book values, dividends and various value-relevant indicators as implicit functions of the forecast post-horizontal abnormal earnings growth rate, finishes the section. Section III concludes the paper.

I. Theoretical analysis

A. The abnormal earnings model

A brief description of the abnormal earnings model is provided below. According to established financial theory, the value of a firm's equity equals the sum of its expected dividends over an infinite horizon discounted to present time (Williams (1938)):

$$v_0 = \sum_{t=1}^{\infty} \frac{d_t}{(1+r_t)^t} \qquad \text{DDM (1)}$$

where v_0 = present value of firm at the end of year 0,

d_t = net dividends[2], and

r = cost of equity capital (; here assumed constant, which, if the cost of debt and the return on total assets (at market values) are assumed fixed, implies constant market valued leverage.).

[2] Net dividends = transactions with stockholders dividends + stock repurchases - stock issues.

The abnormal earnings model is derived from the dividend discount model under the assumption of a clean surplus relation (CSR) among earnings, book values of equity and dividends:

$b_t = b_{t-1} + e_t - d_t,$ CSR (2)

with e_t = earnings end of year t; and

b_{t-1} = book value of equity at end of year $t-1$.

Over a finite time-horizon S, the sum of discounted dividends in (1) could be rearranged using the CSR in the following manner:

$$\sum_{t=1}^{S}\left(\frac{d_t}{(1+r)^t}\right) = b_0 + \sum_{t=1}^{S}\left(\frac{e_t - rb_{t-1}}{(1+r)^t}\right) - \frac{b_S}{(1+r)^S}. \quad (3)$$

If S is let to approach infinity and if it is supposed as in Ohlson (1995) that $\frac{b_S}{(1+r)^S}$ then vanishes (i.e., it is assumed that the growth in book value is less than r), we arrive at the abnormal earnings model. According to this model, the present value of the firm's equity is equal to current book value of equity plus the discounted infinite stream of future abnormal earnings:

$$v_0 = b_0 + \sum_{t=1}^{\infty}\frac{e_t - r_t b_{t-1}}{(1+r)^t} = b_0 + \sum_{t=1}^{\infty}\frac{a_t}{(1+r)^t} \quad \text{AEM (4)}$$

where $a_t = e_t - r_t\, b_{t-1}$, the abnormal earnings of year t,

r = cost of equity capital, assumed constant for all periods t (holds true if the market valued debt-to-equity ratio, return to assets and to debt all are constant).

In practice, it is uncommon to produce explicit forecasts of abnormal earnings to infinity as is implicit in (4). Instead, a model of terminal value is employed for periods beyond a horizon S. In AEM valuations it is often assumed that the abnormal earnings of year S, a_S, grow at a constant rate of g in perpetuity from year S and onwards. Then the following model is obtained:

$$v_{AEM} = b_0 + \sum_{t=1}^{S} \left(\frac{a_t}{(1+r)^t} \right) + \frac{a_S (1+g)}{(1+r)^S (r-g)}$$
$$= b_0 + \sum_{t=1}^{S-1} \left(\frac{a_t}{(1+r)^t} \right) + \frac{e_S - r b_{S-1}}{(1+r)^{S-1} (r-g)} \tag{5}$$

where the rightmost term is the terminal value, which represents the present value of the perpetual stream of constantly growing abnormal earnings of year *S*; the terminal value expression is only valid if $r>g$. CT use analysts earnings forecasts in conjunction with the CSR to explicitly predict abnormal earnings for years one to five, that is they use S=5. CT assume that 50% of earnings is paid out as dividends each period, i.e., $d_t = p\ e_t$ with p=0.5, and that book value thus is updated as $b_{t+1} = b_t + e_{t+1} - p\ e_t$, and that in the terminal value calculation the abnormal earnings of year +5 are applied.

B. Implications of constant growth of abnormal earnings in perpetuity

The implications of letting abnormal earnings grow at constant rate for infinite time after a horizon are perhaps not so well appreciated. Hereunder we analyze what such a terminal or continuing value model implies about the post-horizontal and asymptotic behavior of dividends, earnings, book values, return-on-equity, price-to-book and price-to-earnings ratios.

Constant perpetual growth in abnormal earnings simply means that the difference between earnings and r times opening book value the next period is (1+g) times larger than in the current period, i.e., $e_{t+1} - r\ b_t = (1 + g)\ (e_t - r\ b_{t-1})$. By assumption, book values, earnings and dividends obey the clean surplus relation, that is $b_{t+1} = b_t + e_{t+1} - p\ e_t$.

Let r be a positive constant and p and g constants such that $0<p\leq 1$ and $-1<g<r$. Then the following system of difference equations describes how earnings and book value evolve over time

$$e_{t+1} - r\ b_t = (1 + g)\ (e_t - r\ b_{t-1}), \tag{6}$$
$$b_{t+1} = b_t + e_{t+1} - p\ e_t.$$

Solving[3] for b_t and e_t as functions of discrete time and with initial values equal to b_0 and e_1, results in

$$e_t = \frac{g(1+g)^{t-1}(e_1 - r\,b_0) + (g\,b_0 + e_1(p-1))r(1+r-pr)^{t-1}}{g-(r-pr)}, \quad t=1..n, \text{ and} \tag{7}$$

$$b_t = \frac{-(1+g)^t(p-1)(e_1 - r\,b_0) + (g\,b_0 + e_1(p-1))(1+r-pr)^t}{g-(r-pr)}, \quad t=0..n, \tag{8}$$

with the condition $g \neq r - pr$ preventing zero denominators. Dividends each post-horizontal period are then straightforwardly expressed as $p\,e_t$ and abnormal earnings as $e_t - r\,b_{t-1}$, where the latter quantity by construction is equal to $(e_1 - r\,b_0)(1+g)^{t-1}$; t= 1..n. It should be clear, from (7) and (8), that constant growth in abnormal earnings implies a non-trivial behavior of post-horizontal earnings, dividends and book values. Using (7) and (8) together with the AEM and the DDM enables interesting theoretical and empirical analyzes.

Without going into details, we note the following related to (7) and (8):

- Inspection of (7) and (8) reveals that the respective asymptotic growth rates of e_t and b_t converge to either the retention rate r–pr or g depending on what term is larger, i.e.:

  Asymptotic growth rate of e_t and b_t: Max(g, $r - pr$).

- CT write (p. 1664) that other studies suggest that, under so-called conservative accounting, return-on-equity or *ROE*, e_t/b_{t-1}, will converge to r as t increases towards infinity, but, at least in the framework employed here, perhaps as opposed to conservative accounting then, this is only true for r-pr>g. When g>r-pr, e_t/b_{t-1} will converge to $g/(1-p)$.[4] Thus:

  Asymptotic return-on-equity (*aROE*): $\begin{cases} r, & \text{for } r - pr > g \\ g/(1-p), & \text{for } r - pr < g. \end{cases}$

[3] Solution in Appendix A.

[4] See Appendix B for details.

- When dividends are predicted to infinity as $p\ e_t$, with e_t given by (7), and that stream is discounted to get present value, the value of p – "the dividend policy" – does not affect *ceteris paribus* the present value of the stream. In other words: the rate of wealth distribution does not determine value - which is in vein with the Miller-Modigliani dividend irrelevancy proposition. What p does is that it governs the number of periods it takes for the discounted sum to converge; larger p means faster convergence.
- Values of g in the interval $-1<g<0$ could be interpreted to represent the so-called Ohlson (1995) linear information dynamics model (LIM) with the coefficient on abnormal earnings equal to $(1+g)$ and the coefficient on "other information" and the error term both equal to zero.
- Negative abnormal earnings of year S may cause e_t and b_t to approach negative infinity as t grows large.

CT (p. 1637) argue that the AEM offers an advantage over the GDM in that it can be used to compute various diagnostics such as future return-on-equity, price-to-earnings ratios and price-to-book ratios, which can be used to verify the realism of the forecasts and their implications. (As noted above, this can also be done with the GDM.) Without explicit formulas for the behavior of the quantities involved, iterative procedures are however required for their computation. The behavior of return-on-equity is obtained more or less directly from the formulas for earnings and book value as shown above, while the behavior of price-to-earnings and price-to-book ratios is dependent on the future evolvement of price. An equation for future price must therefore be derived. Post-horizontal price (or value) v_t will evolve as

$$v_{t+1} = v_t\,(1 + r) - p\ e_{t+1}\ , \qquad (9)$$

with e_t given by (7). The initial value of v, v_0, is needed in the analysis. Since abnormal earnings are assumed to grow at g in perpetuity, v_0 can be obtained as the present value of a constantly growing perpetuity, or by solving (5) for S=1, i.e.:

$$\begin{aligned} v_{AEM} &= v_0 = b_0 + \sum_{t=1}^{S-1}\left(\frac{a_t}{(1+r)^t}\right) + \frac{e_S - rb_{S-1}}{(1+r)^{S-1}(r-g)} \\ &= b_0 + \frac{e_1 - rb_0}{r-g} \\ &= \frac{e_1 - gb_0}{r-g}. \end{aligned}$$

Solving[5] (9) for v_t as a function of time, for t=0..n, with initial values e_1 and $v_0 = (e_1 - g\, b_0) / (r - g)$, results in

$$v_t = \frac{g(1+g)^t\, p(e_1 - r\, b_0) + (g\, b_0 + e_1(p-1))(r-g)(1+r-pr)^t}{(r-g)(g+r(p-1))}. \qquad (10)$$

As t increases towards positive infinity, the ratios v_t/e_t and v_t/b_t converge to:[6]

Asymptotic price-to-earnings (aP/E) ratio: $\begin{cases} 1-p+1/r, & \text{for } r-pr > g \\ p(1+g)/(r-g), & \text{for } r-pr < g. \end{cases}$

Asymptotic price-to-book (aP/B) ratio: $\begin{cases} 1, & \text{for } r-pr > g, \\ \dfrac{pg}{(1-p)(r-g)}, & \text{for } r-pr < g. \end{cases}$

Like the converging return-on-equity ratio, the converging price-to-earnings and price-to-book ratios, each also takes either of two forms depending on the relative magnitudes of the retention rate and the abnormal earnings growth rate.

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[5] For brevity no solution is presented, but it is straightforward to verify that the solution satisfies the equations. Also see the solution to (5), which is solved in a similar manner as (9).

[6] The derivations are akin to the one for *ROE* and are therefore not presented.

C. Invariant equity premia and the equivalence of the AEM and DDM

CT estimate the equity premium with the GDM and obtain a significantly different estimate than with the AEM. At a glance, it might appear disturbing that the premia obtained with the GDM differ substantially from the ones obtained with the AEM. The GDM is after all a special case of the DDM, and the DDM and the AEM are equivalent. Applied the way CT apply them, and CT are careful to point this out, the GDM and AEM are not equivalent, and the GDM approach uses only a subset of the information used in the AEM approach. In particular, apart from utilizing only a subset of available information, the GDM assumes a constant perpetual growth rate of dividends. So, if one uses the AEM and the terminal value model that CT use, the growth rate of dividends in perpetuity is by construction (generally) non-constant, and therefore impossible to model over time with a constant growth rate. The different premium estimates should thus not come as a surprise.

Nevertheless, the DDM and the AEM are equivalent (when applied consistently), so, for completeness, we show analytically that if the DDM is applied to the infinite dividends stream implied by the AEM, the premium estimates coincide. In particular, it is demonstrated that present value according to the DDM and the AEM, respectively, is the same when the CSR is fulfilled, and when there is a single discount rate. CT equate observed market value with present value according to the AEM and the GDM and solve for the discount rate from which they subtract the risk-free rate to get the risk premium. They obtain different premia from either model since the forecasted dividends stream is different from or inconsistent with the one implied by the forecast of abnormal earnings.[7] Since the AEM and the DDM are

[7] Lundholm and O'Keefe (2001) discuss a similar issue, but they consider a case balanced on a knife-edge where "everything" (a, e, b, and d) grows at g beyond the horizon. This can be obtained by setting the payout ratio $p=[e_S-(1+g)b_{S-1}+b_{S-1}]/e_S$, but might, however, result in strange payout ratios, among other things; Lundholm and O'Keefe's post horizon payout ratio is 0.8, which is around 10 times higher than within the horizon.

equivalent when the dividends and abnormal earnings forecasts are consistent, the discount rate – and hence the risk premium - solved for must be same for either model.

As mentioned CT use the equation (5) for value where S denotes the last year for which there are explicit forecasts:

$$v_{AEM} = b_0 + \sum_{t=1}^{S}\left(\frac{a_t}{(1+r)^t}\right) + \frac{a_S(1+g)}{(1+r)^S(r-g)}.$$

The corresponding DDM with dividends expressed as a fraction p of earnings is

$$v_{DDM} = \sum_{t=1}^{S}\left(\frac{d_t}{(1+r)^t}\right) + \sum_{t=S+1}^{\infty}\left(\frac{d_t}{(1+r)^t}\right) = \sum_{t=1}^{S}\left(\frac{p\,e_t}{(1+r)^t}\right) + \sum_{t=S+1}^{\infty}\left(\frac{p\,e_t}{(1+r)^t}\right). \tag{11}$$

We want to show that $v_{DDM} = v_{AEM}$. For any finite horizon S the following relation based on (3) exactly applies:

$$\sum_{t=1}^{S}\left(\frac{d_t}{(1+r)^t}\right) = \sum_{t=1}^{S}\left(\frac{p\,e_t}{(1+r)^t}\right) = b_0 + \sum_{t=1}^{S}\left(\frac{a_t}{(1+r)^t}\right) - \frac{b_S}{(1+r)^S}. \tag{12}$$

Evaluation of the infinite sum in (11) with e_t given by (7), results in

$$\sum_{t=S+1}^{\infty}\left(\frac{p\,e_t}{(1+r)^t}\right) = \frac{b_{S-1}g(1+r) - e_S(1+g\,p+r-p\,r)}{(1+r)^S(r-g)}. \tag{13}$$

Rearrange the right hand side of (13) and apply the CSR to obtain

$$\sum_{t=S+1}^{\infty}\left(\frac{p\,e_t}{(1+r)^t}\right) = \frac{b_{S-1}g(1+r) - e_S(1+g\,p+r-p\,r)}{(1+r)^S(r-g)}$$
$$= \frac{b_{S-1} + e_S - pe_S}{(1+r)^S} + \frac{(e_S - rb_{S-1})(1+g)}{(1+r)^S(r-g)},$$

which simplifies to

$$\sum_{t=S+1}^{\infty}\left(\frac{p\,e_t}{(1+r)^t}\right) = \frac{b_S}{(1+r)^S} + \frac{a_S(1+g)}{(1+r)^S(r-g)}. \tag{14}$$

Finally, evaluate v_{DDM} = (12) + (14) to see that $v_{DDM} = v_{AEM}$.

$$
\begin{aligned}
v_{DDM} &= \sum_{t=1}^{S}\left(\frac{p\,e_t}{(1+r)^t}\right) + \sum_{t=S+1}^{\infty}\left(\frac{p\,e_t}{(1+r)^t}\right) \\
&= b_0 + \sum_{t=1}^{S}\left(\frac{a_t}{(1+r)^t}\right) - \frac{b_S}{(1+r)^S} + \frac{b_S}{(1+r)^S} + \frac{a_S(1+g)}{(1+r)^S(r-g)} \\
&= b_0 + \sum_{t=1}^{S}\left(\frac{a_t}{(1+r)^t}\right) + \frac{a_S(1+g)}{(1+r)^S(r-g)}.
\end{aligned}
$$

Value according to the AEM, applied as CT apply it, is obviously equal to value obtained using the DDM with the dividends stream implied by the terminal value. Hence, the risk premium could be estimated by use of either the AEM or the DDM, though it seems quite unlikely that anyone explicitly forecasts dividends the way that is implied by the assumption of constantly growing abnormal earnings in perpetuity.

II. Empirical analysis

The first part of the empirical analysis concerns the long-term behavior of earnings, book values, dividends and key financial ratios based on CT's forecasts of abnormal earnings growth in perpetuity. The second part considers the general relationship between the discount rate, the risk premium, earnings, book values, dividends, key financial ratios, and the post-horizontal growth rate of abnormal earnings. This analysis also provides upper and lower bounds on the risk premium imposed by the algebra of the valuation model and the restriction that the equity premium be non-negative.

A. Long-term behavior of earnings, book values, dividends and key financial ratios

We use the U.S. data analyzed by CT and apply the derived formulas (7) and (8) to the very same forecasts of growth in abnormal earnings to obtain the long-term behavior for dividends, earnings and book values; the main focus of the analysis is on their post-horizontal behavior. We also check - roughly - how this implied future behavior compares to available historical evidence. A problem with a comparison of this kind is to find a historical sample that is similar enough to CT's sample in terms of firms and aggregation method.

The choice of growth rate of abnormal earnings in perpetuity is crucial in the determination of the size of the risk premium. CT apply positive abnormal earnings growth rates (which they mention that they consider high) to positive a_S. A historical record of abnormal earnings growth for a capitalization-weighted aggregate such as CT's would have been a useful reference point. For the 37-year period 1963-1999, Penman and Nissim (2001) (PN henceforth) estimate across U.S. firms[8] average (median) year-to-year growth rates of abnormal earnings to -56.8% (-11.8%).[9] Empirical evidence in the framework of the Ohlson (1995) linear information dynamics model, e.g., Dechow et al (1999) and Mcrae and Nilsson (2002), suggests that (deflated) abnormal earnings for individual firms on average are mean-reverting, i.e., $1+g<1$; Dechow et al (1999), for instance, find that $1+g=0.62$, so $g= -0.38$. All these estimates are sensitive, of course, to the particular cost of capital assumed. Nevertheless, such low growth rates applied to CT's aggregate data would lead to negative discount rates and risk premia. Here we are however dealing with a capitalization-weighted aggregate of firms, and a somewhat daring hypothesis supporting a positive g for the aggregate would be that large-capitalization firms earn monopoly (accounting) rents, while most individual, smaller-capitalization firms face non-positive gs. This, then, would force g for the capitalization-weighted aggregate to be positive.

In addition, the path to convergence (its length and direction of approach) depends on p. For CT's sample it takes many periods until (approximately full) convergence in earnings and

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[8] The sample includes the NYSE and AMEX firms which use U.S. GAAP and which are present in the combined COMPUSTAT (Industry and Research) files.

[9] PN use for book value the average of the opening and closing book value each year, and assume a cost of equity equal to the one-year treasury rate plus 6%. PN exclude observations of ratios where the denominator is non-positive. This might bias the abnormal earnings growth rate downwards, because PN find abnormal earnings mean-reverting, so firms with negative abnormal earnings should have an increased likelihood of exhibiting positive abnormal earnings growth.

book value growth is reached. Therefore, comparisons to historical evidence gathered over limited time-periods, say 80 years, may be awkward to do.

< Table I around here >

Let us examine the behavior of future earnings given the forecast stated as of April 1985. Table I (which is an amalgamation of key figures in Table I and II in CT) contains actual (reported) earnings and book values as of year 0, that is for fiscal year 1984, as well as forecast earnings and corresponding book values within the five-year explicit forecast horizon, i.e., for years +1 to +5, that is, for fiscal years 1985 to 1989. In the analyses to come, (actual and forecast) earnings for 1984 to 1988, that is, e_0, e_1, ..., e_4, will be taken from Table I. Forecast earnings from 1989 and onwards, that is, e_5, e_6, ..., e_n, are obtained by applying (15), which is a restated version of (7), to the forecasts and parameter values (r, g, p, e_5, b_4) of the April 1985 forecast:

$$e_t = \frac{g(1+g)^{t-5}(e_5 - r\, b_4) + (g\, b_4 + e_5(p-1))r(1+r-pr)^{t-5}}{g-(r-pr)}, \qquad t=5..n. \qquad (15)$$

Inserting r, g, p, e_5, and b_4 of the forecast of April 1985, yields

$$e_t = \text{-}39057 \text{ x } 1.0719^t + 226161 \text{ x } 1.0843^t, \qquad t=5..n.$$

For 1989, the last year within the horizon, use t=5 to obtain the value of forecast earnings e_5 as 283706. For 1990 (t=6), earnings e_6 become 308308. As t grows large, the left term with the negative sign will become insignificant relative to the right one, and the growth rate of earnings will approach 8.43%. The dividends behavior is obtained by multiplying earnings by the payout ratio of 0.5 as assumed by CT. Book values for 1984 to 1987, that is, b_0, b_1, b_2, and b_3, are taken from Table I. Book values from 1988 and forward, i.e., b_4, b_5,..., b_n, are obtained by applying (16), a restated version of (8), presented below to the r, g, p, e_5, and b_4 of the April 1985 forecast:

$$b_t = \frac{-(1+g)^{t-4}(p-1)(e_5 - r\,b_4) + (g\,b_4 + e_5(p-1))(1+r-pr)^{t-4}}{g-(r-pr)}, \quad t=4..n. \quad (16)$$

< Table II a) around here >

Table II a) depicts the period-to-period earnings (and dividends) growth rate, i.e., e_t/e_{t-1}, t=1..n. The earnings growth rate year +1 vary considerably, from 9.7%, for the forecast stated in 1998, to 32.7%, for the forecast stated in 1988. The marginal earnings growth rates for year +5 are in the range 11.2%-14.4% and thus less dispersed. The arithmetic average year-to-year earnings growth rate within the horizon is 14%, which appears high. Beyond the explicit forecast horizon, the growth rate of earnings and dividends decreases with time towards a constant growth rate. The rate of convergence is rather slow; for this sample, it takes several hundred periods for the growth rate to converge (practically). Overall, the implied post-horizontal behavior of earnings and dividends looks rather reasonable. For the first year in the post-horizontal periods, growth rates of earnings are between 5% and 9%. The converging rates lie between 4% and 8% with an average of 5.6%. However, if forecasts stated in 1985 and 1998 are excepted, the converging growth rates vary between 5% and 7%. The average of the arithmetic average growth rates for the first 80 years beyond our forecast horizons is 6.4%. For the first 80 years including the 5-year period with explicit forecast, the average of average earnings growth rates is 6.9%. These figures appear quite reasonable if compared to the average "growth in S&P earnings" which has been 6.6% since the 1920s, according to CT (who cite Wall Street Journal).[10]

< Table II b) around here >

[10] PN do not report any statistics on earnings or dividends behavior, which we think - despite that at least dividends in a sense are redundant given the AEM approach used by PN - is unfortunate, since they are important in their own right.

Table II b) concerns the behavior of book value of equity using the data in CT. Book value year-to-year growth defined as b_t/b_{t-1}, t=1..n, increases within the explicit forecast period, for any given year. The average marginal growth rate is 7.7% year +1, and 9.1% year +5. Beyond the explicit forecast horizon, the pattern is similar to that for growth rates of earnings: book value growth rates are between 8% and 10% in the beginnings of the post-horizontal periods, but decrease with time and the converging growth rates lie between 4% and 8%. Again, the forecasts made in 1985 and 1998 have the most extreme implications. For the 37-year period 1963 to 1999, PN estimate the average (median) year-to-year growth in book value of equity to 11.3% (9.0%). In CT's sample the average across years of the (arithmetic) averages of growth rate of book value over the first 37 years is 7.8% (regardless of whether opening, or as in PN, the average of opening and closing book value are used). PN's average and median growth rates are not for an aggregate of U.S. firms, but an average and median of individual firms' equity growth rates.[11] Apart from that the samples are different, one reason for the discrepancy of the numbers might be that we have used a dividend payout ratio of 50% for the aggregate earnings, while other payout ratios probably apply for the individual firms. Furthermore, smaller-capitalization firms might grow their equity faster than larger firms, and that would increase an equal-weighted average relative a capital-weighted aggregate such as CT's.

< Table II c) around here >

Table II c) documents the behavior of profitability measured as return-on-equity, computed as year-end earnings divided by the opening value of equity, e_t/b_{t-1}, t=1..n. The average return-on-equity the first year in the explicit forecast horizon 15.3%. The minimum and maximum are 13.2% and 17.0%, respectively. The average return-on-equity in the explicit forecast period is 16.9%, evidencing that for a given forecast, return-on-equity

[11] As mentioned, PN delete observations of ratios where the denominator is non-positive.

generally increased after the first year year. For the first post-horizontal year, for all forecasts stated, the return-on-equity is between 17% and 20%. It then declines to long-term convergent values that vary from below 9% to 18%; if 1985 and 1998 are excepted, the range is 10.12-12.75%. Penman and Nissim find that the average (median) return-on-equity across firms for 1963 to 1999 is 10.2% (12.2%). In CT's sample, the average of the average for the first 37 years including the explicit forecast period is 15.6%, and for years +5 to +41, it is 15.1%. Penman and Nissim divide year-end earnings by the average of opening and closing book value of equity, while opening equity is used here. The convention used here should yield relatively higher return-on-equity ratios, since opening equity is expected to be lower than average equity, for book value is increasing over time. Indeed, in unreported calculations, based on the same convention as PN, the respective averages become 15.1% and 14.6%. However, as mentioned PN's figures are not directly comparable to ours, because for one thing they do not reflect an aggregate of firms. The implied post-horizontal return on equity seems for most years rather high compared to the historical evidence (to the extent they are comparable).

< Table II d) around here >

For completeness, results are provided for the asymptotic behavior of *P/E* and *P/B* ratios given CT's forecasts. Table II d) shows the behavior of the *P/E* ratio. The initial *P/E* ratios vary between 11 and 24, the mean being 15.6. The most extreme observations are associated with the forecasts of 1985 and 1998. The *P/E* ratios at the end of the explicit horizon are in the range 9.42-16.12. Beyond the horizon, *P/E* ratios decline rather slowly, and the mean asymptotic ratio is 9.8.

< Table II e) around here >

Table II e) contains the results for the *P/B* ratio. The *P/B* ratio is higher year +5 than in year +1 for the forecast of 1988, otherwise the *P/B* ratio decreases within the horizon. Within the horizon, the *P/B* ratios are highest for the forecast produced in 1998 and lowest for the one of 1985. Beyond the horizon, *P/B* ratios decline towards one, except for the forecast of 1985, for which the ratio converges to 1.42.

B. The general relationship between risk premia and growth of abnormal earnings in perpetuity

The above analysis is extended by consideration of the general relationship between the discount rate, the risk premium and the post-horizontal growth rate of abnormal earnings. This analysis also provides upper and lower bounds on the risk premium imposed by the algebra of the valuation model and the restriction that the equity premium be non-negative. For any given year, CT use a single forecast of *g*, equal to the risk-free rate minus 3%, for which they compute the corresponding implied discount rate and risk premium. As a matter of fact, however, the valuation equation (5) provides a "functional" relationship between *g* and *r* and *r-f*. Below an analysis of that relationship is provided. This will, among other things, shed light on the robustness of risk premium estimates with respect to varying assumptions about the growth rate of abnormal earnings.

It is not possible to solve (5) explicitly for *r* and the risk premium (which is obtained by subtracting from *r* the constant risk-free rate). However, we can solve it for *g* and analyze *r* as well as the equity premium as implicit functions of *g*. Thus, solving (5) for *g*, yields:

$$g = r + \frac{e_S - rb_{S-1}\left(b_0 - v_{AEM} + \sum_{t=1}^{S-1} a_t\right)^{-1}}{(1+r)^{S-1}}, \qquad (17)$$

and setting S=5, results in

$$g = \frac{pr\left[(1+r)\left[(1+r)[(1+r)e_1 + e_2] + e_3\right] + e_4\right] + e_5 - (1+r)^4 v_{AEM}}{b_0 + \left[1 + pr(3 + r(3+r))\right]e_1 + \left[1 + pr(2+r)\right]e_2 + e_3 + pre_3 + e_4 - (1+r)^4 v_{AEM}}. \qquad (18)$$

In addition, the algebra of the AEM model, i.e., equation (5), and the restrictions on r and g used in the derivation of the terminal value together with the "common sense" restriction that the risk premium be non-negative given a specified risk-free rate, actually provide upper and lower bounds on the "permissible" range and domain for r and r-f and g, respectively, viewing r and the risk premium as "functions" of g. Knowledge about the specific values of those bounds should be helpful in drawing conclusions about what a reasonable range for the equity risk premium is.

The results of the analysis, for all years and with forecasts and parameter values taken from Table I are presented in Table III. To facilitate understanding of Table III, the analysis for 1986, a rather typical year, is presented in some detail. That discussion is supported by Figure 1 which gives the main results that year in graphical form including the relationships between g and the discount rate r, and between g and the risk premium r-f, as well as the upper and lower bounds on the risk premium. It is also possible to read off from Figure 1 the converging growth rates of earnings, dividends and book values that a particular forecast of g results in.

< Table III around here >

< Figure 1 around here >

The relationship defined by (18) between r and g for 1986 is plotted (the dashed curve) in Figure 1 with r displayed on the vertical axis and g on the horizontal. As can be seen, r increases as g increases, and at an increasing rate. The risk premium as a function of g is easily obtained by subtracting the constant risk-free rate, f, from the discount rate, r. The risk premium plots as a solid curve and is labeled r-f.

The valuation equation (5) was derived under the assumption that $r>g$, therefore r and g will have a common limiting upper value *rgmax*, i.e., r, g<*rgmax*,. We can find *rgmax* by setting g equal to r and solve for r in (16). *rgmax* is in the figure represented by two thin lines, one vertical and one horizontal, marking the upper bounds for *r* and g, respectively. By subtracting from *rgmax* the risk-free rate, *f*, the upper bound on the risk premium, *rgmax-f*, is found. *rgmax-f* plots as a thin horizontal line. In 1986 *rgmax* equals 17%, therefore the maximum values of *r* and *g* are just below 17%. As the risk-free rate, *f*, in 1986 is 7.3%, the risk premium, *r-f*, cannot not equal or exceed 9.7% (i.e., *rgmax* minus *f*). Inversely then, a risk premium of just below 9.7% implies a discount rate and a post-horizontal growth rate for abnormal earnings of approximately 17%.

Graphed as a thin vertical line is *gmin*, which is the value of *g* that corresponds to a risk premium equal to zero; *r* is set equal to the risk-free rate and inserted in (18) and the resulting growth rate is labeled *gmin*. It is probably not controversial to view this as a lower bound on *g*. For 1986, an abnormal earnings growth rate of around –10% yields a zero equity premium.

As mentioned, a few studies have documented quite low growth rates of abnormal earnings, around –40%. For reference, *rmin*, an even lower bound on *r* than the one posed by the risk-free rate, is calculated here by setting the denominator of (18) equal to zero and solving for *r*. The largest positive real root to that equation is then used as *rmin*. (This is the asympote along which *g* viewed as a function of *r* approaches negative infinity when *r* approaches *rmin* from above.) *rmin* plots as a thin horizontal line. For 1986 *rmin* is equal to -2.6%, so no matter how small *g* is assumed to be, *r* will always stay above *rmin*. Very negative growth rates for the aggregate market are of course peculiar, and appear to bring about negative discount rates in this sample, but in 1985 *rmin* was 4.6% indicating that even very negative growth rates may correspond to positive discount rates; the equity premium was negative though.

For a given change in *g*, how much will *r* and the equity premium, *r-f*, change? To answer this we need the derivative *dr*/*dg*. Since (18) cannot be solved explicitly for *r*, neither can *dr*/*dg* be computed directly. Instead we proceed as follows to find *dr*/*dg*. Given (18) and adopting the notation g=g(r), i.e., g as a function of r (treating all other variables as constants), then, if $g(r)$ is monotonic in a certain domain, then there exists in the same domain an inverse function $r = g^{-1}(g)$ with derivative dr/dg. For a function and its inverse, the derivatives relate as dr/dg = 1/(dg/dr). And as dg/dr can be computed directly from (18), dr/dg is easily obtained. Computing dr/dg for *gmin* and *rgmax* gives us, since in CT's sample $r = g^{-1}(g)$ is a function that increases at an increasing rate, the minimum and maximum rates of change, respectively. The values for 1986 are 0.21 and 0.56, i.e., the sensitivity of the discount rate and the risk premium (since it is equal to r minus a constant f) to a given change in g is almost 2.7 times higher when the growth rate is equal to *rgmax* than when it is equal to *gmin*.

The asymptotic growth rate of earnings, dividends and book values, *asympg*, is, as was pointed out earlier, equal to Max(g, $r - pr$), i.e., equal to the larger of the growth rate of abnormal earnings g or the retention rate $r - pr$. *asympg* is displayed in Figure 1 as a dash-dot curve. We mark the "balance point" (the point of equivalence), that is where $g = r - pr$, by a thin solid vertical line crossing the horizontal axis where $g = r - pr$. To the right of this line, for larger values of g, the asymptotic growth rate for earnings, dividends and book values is equal to g. This is so because (in this sample) the derivative of $r - pr$ with respect to g is increasing but smaller than one, and therefore $r - pr$ will be smaller than g when g is increased beyond the point where they are equal; a one unit increase in g corresponds to less than one unit's increase in $r - pr$. To the left of this point, i.e., for smaller values of g, a one unit's decrease of g implies less than a unit's decrease in $r - pr$, which then means that $r - pr$ will be larger than g on that side. In 1986, the "balance point" is equal to 5.9%. Let us return to the discussion concerning the implications from a risk premium of just below 9.7% (=*rgmax*-f). Since the g that corresponds to this risk premium is around 17%, it means that the g in question is well to the right of the "balance point". This then means that the converging growth rates of earnings, dividends and book values also will be equal to *g*, i.e., around 17%. For 1986, in addition to implying a discount rate of 17% and an abnormal earnings growth rate of 17%, it thus appears that a risk premium near 9.7% also has quite extreme implications in terms of growth in earnings, dividends and book values.

Let us recapitulate CT's findings for 1986. As of April 1986, they forecasted *g*, the post-horizontal growth rate for abnormal earnings, to be 4.3%. That forecast thus, according to the figure, corresponds to a discount rate of 11.3%, a risk premium of 4%. Since *r-pr* is equal to 5.6% and hence larger than *g*, the converging asymptotic growth rate *asympg* for earnings, dividends and book values equals 5.6%.

< Figure 2 around here >

Figure 2 and Table III present the relationship in 1986 between *g* and *r* and asymptotic return-on-equity (*aROE*). On the *r-g* coordinate plane is a projection of the *aROE* graph from the *aROE* direction (; this is the same *r-g* curve as in Figure 1). The kink of the graph appears where *r-pr=g*; that is for *r=rbal*=11.9% and *g=gbal*=5.9%. At this point *aROE* would be equal to *rbal*=11.9%, or to *aROE=gbal*/(1-*p*)=0.059/(1-0.5)=11.9%, but *r-pr=g* was precluded in the solution. When *r-pr>g* then *aROE=r*, and when *g>r-pr*, *aROE=g*/(1-*p*)=2*g*. The maximum *aROE* is given by *rgmax* and is equal to *aROEmax=gmax*/(*1-p*)= *rgmax*/(*1-p*)=0.17/0.5=34%. The *aROE* that corresponds to a risk premium of zero is found when *r=f*=7.3%, and then *aROE=r*=7.3%.

< Figure 3 around here >

Figure 3 and Table III show for 1986 the relationships between *r*, *g* and the asymptotic *P/B* (*aP/B*) ratio, and between *r*, *g* and the asymptotic *P/E* (*aP/E*) ratio, respectively. When *r-pr<g*, and *g* and *r* approach *rgmax, aP/E* and *aP/B* ratios both approach positive infinity. For *r-pr>g*, the *aP/B* ratio is constant one. For *r-pr<g*, the *aP/B* ratio is $pg/[(1-p)(r-g)]$. As *g* increases, *r* increases, but at a slower rate, therefore, as *r-g* approaches zero from above, *aP/B*

increases towards positive infinity. For $r\text{-}pr<g$, as g and r decrease towards to the point where $r\text{-}pr=g$, aP/B approaches one from above. For $r\text{-}pr<g$, the aP/E ratio equals $p(1+g)/(r\text{-}g)$. As g increases, r increases, but at a slower rate. Hence, the aP/E ratio increases, as g increases. For $r\text{-}pr>g$, the aP/E ratio equals $(1\text{-}p+1/r)$. Thus, as r (and g) decrease, the aP/E ratio increases. The aP/E ratio corresponding to a zero risk premium is obtained when $r=f$ =7.3%, and is equal to $(1\text{-}p+1/r)$= (0.5+1/0.073)=14.2. Regardless of the direction of approach, as r and g get closer to the point where $r\text{-}pr=g$, the aP/E ratio gets smaller, and thus approaches a sort of minimum equal to $(1\text{-}p+1/rbal)$= (0.5+1/0.119)=8.9. So, for a certain part of the permissible domain, the asymptotic *P/E* ratio decreases as r and g increase. The aP/E ratio is hence not a monotone function of r and g.

Turning to Table III, having discussed 1986 in some detail, we notice that many key quantities remain relatively stable across years, excepting 1985 and 1998 which were the years with the highest and the lowest risk-free rate, respectively. For instance, *rgmax* is on average 18.2% and exhibits limited variation. The maximum risk premium, *rgmax-f*, is on average 10.4%, with a low of 6% in 1985 and a high of 15.1% in 1998. The first eight years all have maximum equity premia below 10%. In light of this, standard equity premia of 8-9% stand out as high and in some cases even algebraically impossible.

The sensitivity of the discount rate and the risk premium to changes in g, increases with g. For *gmin*, dr/dg equals 0.29 on average, and for rgmax the derivative is 0.62 on average. Hence, the discount rate and the riskpremium are considerably more sensitive to changes in the growth rate when it is high than when it is low.

The "balance point", the value for which the retention rate $r\text{–}pr$ and g are equal, is on average equal to 5.8%. Given CT's forecasts, it is only in 1985 that g exceeds the retention rate and as a consequence becomes the driver of the converging growth of earnings, dividends and book values. Again, this time in view of the average "balance point" of 5.8%, traditional equity premium estimates of around 8% appear quite high. Given a risk-free rate of around 8% - its mean in CT's sample - a discount rate of 16% is implied. This would imply a growth rate of abnormal earnings *g* of say 14-15% in perpetuity. And since such a high *g* surely exceeds the retention rate *r-pr*, the asymptotic growth rate of earnings, dividends and book values would be equal to *g*, that is equal to 14-15%. Such high growth rates are simply unrealistic for the aggregate market.[12]

Moreover, a *g* of 14-15% implies asymptotic *ROE* of around 28-30%, which appear equally unrealistic. Assuming a *r* of 16% and a *g* of 14% suggests that the asymptotic *P/B* ratio would be $pg/[(1-p)(r-g)]$ or 7, and the asymptotic *P/E* ratio would equal $p(1+g)/(r-g)$ or 28.

III. Concluding summary

The terminal value model of constant growth in abnormal earnings in perpetuity may appear simple, but its implications for earnings, dividends and book values are somewhat intricate. In this paper we presented analytical formulas for earnings, dividends and book values when abnormal earnings grows at a constant rate. The formulas were applied to the data in the paper by Claus and Thomas in which they estimate the U.S. equity risk premium using the abnormal earnings model with constant perpetual growth in abnormal earnings model as a model for terminal value. Several interesting results were obtained. These are summarized below.

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[12]Since the observed payout ratio varies between 34% and 56% in the period 1985-1998, while a payout ratio of 0.5 was used throughout the analysis, the sensitivity of the results with regard to varying assumptions about the payout ratio for periods *beyond* the horizon was checked. For this sample, for a given *g*, the discount rate and the risk premium increase as *p* is increased; and at an increasing rate. However, for p=0.4 and p=0.6, respectively, the discount rate and the risk premium never change more than slightly more than 1% for any permissible value of *g*, relative to their values under the base case of p=0.5.

CT obtain different estimates of the equity premium with the Gordon dividend growth model than with the abnormal earnings model. We showed that equity premium estimates for the dividend discount model and the AEM coincide if the dividend discount model is applied to the dividends stream implied by the forecasted abnormal earnings stream.

We analyzed empirically what CT's assumptions about growth in abnormal earnings implied about the behavior of earnings, dividends, book values, and key financial ratios, and found the implications rather reasonable compared to historical evidence.

The functional relationship between the equity risk premium and the growth rate of abnormal earnings in perpetuity was derived and analyzed, providing among other things upper and lower bounds on the permissible range of the equity premium. We derived explicit formulas for the post-horizontal behavior of the price-to-earnings ratio, the price-to-book ratio and return-on-equity. Based on these formulas, equations were derived for the asymptotic return-on-equity, price-to-earnings and price-to-book ratios. These show that return-on-equity will converge to the discount rate when the retention rate is larger than the perpetual growth of abnormal earnings. Otherwise return-on-equity converges to the abnormal earnings growth rate divided by one minus the payout ratio. The converging price-to-earnings and price-to-book ratios, each also take either of two forms depending on the relative magnitudes of the retention rate and the abnormal earnings growth rate. For the period studied, 1985-1998, traditional equity premium estimates of say 8-9% appear very high – and are in some cases even algebraically impossible. For instance, an equity premium of around 8%, corresponds, given a payout ratio of 0.5 and a risk-free rate of 8%, on average to a growth rate of abnormal earnings of around 14%, which, in turn, imply converging growth rates of earnings, dividends and book values of also around 14%. The corresponding asymptotic return-on-equity would be close to 30%, the asymptotic price-to-book ratio around 7, and the asymptotic price-to-earnings ratio near 30.

Appendix A

Solve the simultaneuos equations (6) below for e_t and b_t.

$$\begin{aligned} e_{t+1} - rb_t &= (1+g)(e_t - rb_{t-1}), \\ b_{t+1} &= b_t + e_{t+1} - p\, e_{t+1}. \end{aligned} \tag{6}$$

Substituting $b_{t-1} = b_t - e_t + p\, e_t$ in the first line above results in

$$e_{t+1} - rb_t = (1+g)(e_t - rb_{t-1}) = (1+g)(e_t - r(b_t - e_t + p\, e_t\ )).$$

Rearrange the first and second line of the system to get

$$\begin{aligned} &e_{t+1} + (1+g)(-1 + r(p-1))e_t + g\ r\ b_t = 0, \\ &(p-1)e_{t+1} + b_{t+1} - b_t = 0. \end{aligned}$$

Find the general solution to the homogeneous equation system above. This yields the full solution, i.e., the functions for e_t and b_t that solve the system.

Try functions of forms

$$e_t = mc^t,\ \ e_{t+1} = mc^{t+1};\ \ b_t = nc^t,\ b_{t+1} = nc^{t+1}.$$

Insert these into the system

$$\begin{aligned} &mc^{t+1} + (1+g)(-1 + r(p-1))mc^t + g\ r\ nc^t = 0, \\ &(p-1)mc^{t+1} + nc^{t+1} - nc^t = 0. \end{aligned}$$

Divide through by c^t (c^t must therefore not be equal to zero)

$$mc + (1+g)(-1 + r(p-1))m + g\ r\ n = 0, \tag{19}$$

$$(p-1)mc + nc - n = 0. \tag{20}$$

State system in matrix notation

$$\begin{bmatrix} 1 & 0 \\ p-1 & 1 \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix} c + \begin{bmatrix} (1+g)(-1 + r(p-1)) & gr \\ 0 & -1 \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix} = 0\,,$$

and rewrite

$$\begin{bmatrix} c + (1+g)(-1 + r(p-1)) & gr \\ c(p-1) & c-1 \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix} = 0.$$

To avoid trivial solutions to the above system, force the coefficient matrix to be singular, i.e., set its determinant to zero, and solve for *c*.

$$\begin{vmatrix} c+(1+g)(-1+r(p-1)) & gr \\ c(p-1) & c-1 \end{vmatrix} = 0$$

The distinct and real roots are:

$$c_1 = 1+g,$$
$$c_2 = 1+r-pr.$$

The non-zero condition on c^t thus implies that $g \neq -1$ and $r \neq -1/(1-p),$ but by virtue of prior assumptions that $0<p \leq 1$ and $r > 0,$ the condition on r reduces to $r > 0.$

Given the roots found, the general solutions e_t and b_t have the following forms[13]

$$e_t = m_1 c_1^t + m_2 c_2^t, \text{ and}$$
$$b_t = n_1 c_1^t + n_2 c_2^t.$$

Relabel the variables in equations (19) and (20), respectively, as

$$m_1 c_1 + (1+g)(-1+r(p-1))m_1 + g\ r\ n_1 = 0, \text{ and}$$
$$(p-1)m_2 c_2 + n_2 c_2 - n_2 = 0.$$

Solve for m_1 and m_2, and substitute c_1=1+g and c_2=1+r-pr, to obtain the solutions

$$m_1 = \frac{gn_1 r}{c_1 + (1+g)(-1+r(p-1)} = \frac{gn_1 r}{1+g+(1+g)(-1+r(p-1)} = \frac{gn_1}{(1+g-p-gp)},$$
$$m_2 = \frac{(1-c_2)}{c_2(p-1)} = \frac{n_2 r}{1+r-pr}.$$

Insert these expressions into the equations for the general solutions for e_t and b_t:

$$e_t = m_1 c_1^t + m_2 c_2^t = \frac{gn_1}{(1+g-p-gp)} c_1^t + \frac{n_2 r}{1+r-pr} c_2^t = \frac{gn_1 (1+g)^t}{(1+g-p-gp)} + \frac{n_2 r(1+r-pr)^t}{1+r-pr}$$
$$b_t = n_1 c_1^t + n_2 c_2^t = n_1 (1+g)^t + n_2 (1+r-pr)^t.$$

[13] See e.g. Goldberg, S (1958, p. 136) Introduction to difference equations. John Wiley & Sons, New York.

Given initial conditions e_1 and b_0, and the system of equations for e_t and b_t above, find values of n_1 and n_2 that solve the system:

$$e_1 = \frac{g n_1 (1+g)^1}{(1+g-p-gp)} + \frac{n_2 r (1+r-pr)^1}{1+r-pr},$$

$$b_0 = n_1 (1+g)^0 + n_2 (1+r-pr)^0.$$

The solutions are

$$n_1 = \frac{e_1 - e_1 p - b_0 r + b_0 pr}{g - r + pr},$$

$$n_2 = -\frac{e_1 - b_0 g - e_1 p}{g - r + pr}.$$

Substitute n_1 and n_2 back in the equations for e_t and b_t to obtain the full solutions:

$$e_t = \frac{g(1+g)^{t-1}(e_1 - r\, b_0) + (g\, b_0 + e_1 (p-1)) r (1+r-pr)^{t-1}}{g + r\,(p-1)},$$

$$b_t = \frac{-(1+g)^t (p-1)(e_1 - r\, b_0) + (g\, b_0 + e_1 (p-1))(1+r-pr)^t}{g + r\,(p-1)}.$$

Note that to prevent from division by zero, the solutions require $g \neq r - p\, r$.

Appendix B

The asymptotic behavior of return-on-equity, e_t/b_{t-1}

By direct application of (7) and (8):

$$e_t = \frac{g(1+g)^{t-1}(e_1 - r\, b_0) + (g\, b_0 + e_1 (p-1)) r (1+r-pr)^{t-1}}{g + r\,(p-1)}, \text{ and}$$

$$b_{t-1} = \frac{-(1+g)^{t-1} (p-1)(e_1 - r\, b_0) + (g\, b_0 + e_1 (p-1))(1+r-pr)^{t-1}}{g + r\,(p-1)},$$

the equation for e_t/b_{t-1} is obtained

$$\frac{e_t}{b_{t-1}} = \frac{\dfrac{g(1+g)^{t-1}(e_1 - r\,b_0) + (g\,b_0 + e_1(p-1))r(1+r-pr)^{t-1}}{g + r\,(p-1)}}{\dfrac{-(1+g)^{t-1}(p-1)(e_1 - r\,b_0) + (g\,b_0 + e_1(p-1))(1+r-pr)^{t-1}}{g + r\,(p-1)}}$$

$$= \frac{g(1+g)^{t-1}(e_1 - r\,b_0) + (g\,b_0 + e_1(p-1))r(1+r-pr)^{t-1}}{-(1+g)^{t-1}(p-1)(e_1 - r\,b_0) + (g\,b_0 + e_1(p-1))(1+r-pr)^{t-1}}$$

As t approaches infinity the ratio becomes:

$$\lim_{t\to\infty} \frac{g(1+g)^{t-1}(e_1 - r\,b_0) + (g\,b_0 + e_1(p-1))r(1+r-pr)^{t-1}}{-(1+g)^{t-1}(p-1)(e_1 - r\,b_0) + (g\,b_0 + e_1(p-1))(1+r-pr)^{t-1}} =$$

$$\lim_{t\to\infty} \frac{g(1+g)^{t-1}(e_1 - r\,b_0)}{-(1+g)^{t-1}(p-1)(e_1 - r\,b_0) + (g\,b_0 + e_1(p-1))(1+r-pr)^{t-1}}$$

$$+ \frac{(g\,b_0 + e_1(p-1))r(1+r-pr)^{t-1}}{-(1+g)^{t-1}(p-1)(e_1 - r\,b_0) + (g\,b_0 + e_1(p-1))(1+r-pr)^{t-1}}$$

If g>$r - pr$ then the $(1+g)^{t-1}$ terms will become dominant as t gets large and the terms $(1+r-pr)^{t-1}$ become relatively insignificant. Therefore:

$$\lim_{t\to\infty} \frac{g(1+g)^{t-1}(e_1 - r\,b_0)}{-(1+g)^{t-1}(p-1)(e_1 - r\,b_0) + (g\,b_0 + e_1(p-1))(1+r-pr)^{t-1}} = \frac{g}{(1-p)}.$$

For the case g<$r - pr$, the terms $(1+g)^{t-1}$ become relatively insignificant. Thus

$$\lim_{t\to\infty} \frac{(g\,b_0 + e_1(p-1))r(1+r-pr)^{t-1}}{-(1+g)^{t-1}(p-1)(e_1 - r\,b_0) + (g\,b_0 + e_1(p-1))(1+r-pr)^{t-1}} = r.$$

The solutions to the difference equations required $g \neq r - pr$, so we need not consider $g = r - pr$.

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Table I

Reproduction of selected key figures from Table I and Table II in Claus and Thomas (2001) (The notation has been modified.)

Market capitalization, book values, dividends, and actual and forecast earnings for US stocks (1985-1998). Forecasts are as of April Year 0. The market consists of firms on the IBES Summary files with forecasts for years +1, +2, and a 5-year earnings growth estimate as of April each year, and actual earnings per share, dividends per share, number of shares outstanding and share prices as of the end of the prior fiscal year (year 0). Book values of equity for year 0 are obtained from Compustat. When missing on the IBES files, forecasted earnings per share for years +3, +4 and +5 are determined by applying the forecasted 5-year growth rate, to year +2 forecasted earnings. *r* is the implied discount rate, that satisfies the valuation relation in equation (21). Abnormal earnings (a_t) equal reported earnings less a charge for the cost of equity (=beginning book value of equity * *r*). Assuming that 50% of earnings are retained allows the estimation of future book values from current book values and forecast earnings. The terminal value represents all abnormal earnings beyond year 5. Those abnormal earnings are assumed to grow at a constant rate *g*, which is assumed to equal the expected inflation rate, and is set equal to *f* ,the current 10-year risk-free rate, less 3%. All per share numbers are multiplied by the number of shares outstanding to get amounts at the firm level, and these are added across firms to get amounts at the market level each year. All amounts, except for dividend payout, are in millions of dollars.

	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998
Panel A. Actual Values for Year 0														
Firms	1559	1613	1774	1735	1809	1889	1939	2106	2386	2784	2965	3360	3797	3673
Book value	1 191 869	1 214 454	1 323 899	1 430 672	1 541 231	1 636 069	1 775 199	1 911 383	2 140 668	2 168 446	2 670 725	3 182 952	3 679 110	3 412 303
Market value	1 747 133	2 284 245	2 640 743	2 615 857	2 858 585	3 143 879	3 660 296	4 001 756	4 918 359	5 282 046	6 289 760	8 207 274	10 198 036	12 908 495
Earnings	154 858	155 201	146 277	167 676	229 070	228 216	218 699	202 275	247 988	290 081	365 079	446 663	547 395	526 080
f (=10 yr rf)	11.43%	7.30%	8.02%	8.71%	9.18%	8.79%	8.04%	7.48%	5.97%	6.97%	7.06%	6.51%	6.89%	5.64%
Panel B. Forecast Earnings for Years +1 to +5														
Year +1	180 945	178 024	186 319	222 497	261 278	257 657	241 760	252 109	295 862	339 694	444 593	512 921	614 932	577 297
Year +2	205 294	203 677	220 178	246 347	284 616	295 321	294 262	308 567	356 086	402 689	518 600	588 001	709 087	682 524
Year +3	228 208	226 018	244 174	273 204	315 204	328 803	328 513	344 742	397 969	450 559	579 954	659 732	800 129	775 707
Year +4	254 181	251 313	271 432	303 642	349 721	366 798	367 521	386 098	445 840	505 315	650 120	742 244	905 787	884 529
Year +5	283 706	280 035	302 529	338 262	388 776	410 028	412 073	433 552	501 081	568 179	730 648	837 577	1 029 061	1 012 294
Panel C. Forecast Book Values for Years +1 to +5 (50% dividend payout ratio)														
Year +1	1 282 342	1 303 466	1 417 059	1 541 921	1 671 870	1 764 898	1 896 079	2 037 438	2 288 599	2 338 293	2 893 022	3 439 413	3 986 576	3 700 952
Year +2	1 384 989	1 405 305	1 527 148	1 665 094	1 814 178	1 912 558	2 043 210	2 191 721	2 466 642	2 539 638	3 152 322	3 733 413	4 341 120	4 042 214
Year +3	1 499 093	1 518 314	1 649 235	1 801 696	1 971 780	2 076 960	2 207 467	2 364 092	2 665 627	2 764 917	3 442 299	4 063 279	4 741 184	4 430 067
Year +4	1 626 183	1 643 970	1 784 951	1 953 517	2 146 641	2 260 359	2 391 227	2 557 141	2 888 547	3 017 575	3 767 359	4 434 401	5 194 078	4 872 332
Year +5	1 768 036	1 783 988	1 936 215	2 122 648	2 341 029	2 465 373	2 597 264	2 773 917	3 139 087	3 301 664	4 132 683	4 853 190	5 708 608	5 378 479
Panel D. Forecast growth in Abnormal Earnings, *g* (= *f* - 3%)														
g (= *f* - 0.03)	8.43%	4.30%	5.02%	5.71%	6.18%	5.79%	5.04%	4.48%	2.97%	3.97%	4.06%	3.51%	3.89%	2.64%
Panel E. Resulting discount rate (*r*) and equity premium (*r-f*).														
r	14.38%	11.27%	11.12%	12.15%	12.75%	12.33%	11.05%	10.57%	9.61%	10.48%	11.03%	9.96%	10.12%	8.15%
r-f	2.95%	3.98%	3.10%	3.44%	3.57%	3.54%	3.01%	3.09%	3.65%	3.50%	3.97%	3.45%	3.23%	2.51%

Table II a)

Forecast year-to-year earnings growth rates

Panel A display marginal earnings growth rates, e_t/e_{t-1}, where t is the number of years into the future relative to origin of forecast. e_0, e_1, ..., e_4, are taken from Table I. e_5, e_6, ..., e_n, are generated by equation (15). Panel B shows average growth rates in different intervals. CT refers to Claus and Thomas (2001) and the figure is the mean.

Panel A. Marginal year-to-year earnings growth rate (%)									
	Year into the future relative to origin of forecast (t)								
Forecasts as of April	1	5	10	20	36	50	80	100	Asymp.
1985	16.8	11.6	8.7	8.6	8.6	8.6	8.5	8.5	8.4
1986	14.7	11.4	6.9	6.7	6.4	6.2	6.0	5.9	5.6
1987	27.4	11.5	7.1	6.9	6.6	6.4	6.1	6.0	5.6
1988	32.7	11.4	7.7	7.4	7.1	7.0	6.7	6.6	6.1
1989	14.1	11.2	8.1	7.8	7.5	7.4	7.1	7.0	6.4
1990	12.9	11.8	7.9	7.6	7.3	7.1	6.8	6.7	6.2
1991	10.5	12.1	7.2	6.9	6.6	6.4	6.1	6.0	5.5
1992	24.6	12.3	6.8	6.5	6.2	6.0	5.8	5.7	5.3
1993	19.3	12.4	6.0	5.7	5.4	5.2	5.0	5.0	4.8
1994	17.1	12.4	6.8	6.5	6.1	5.9	5.6	5.5	5.2
1995	21.8	12.4	7.1	6.7	6.3	6.1	5.9	5.8	5.5
1996	14.8	12.8	6.5	6.1	5.8	5.6	5.3	5.2	5.0
1997	12.3	13.6	6.8	6.4	6.0	5.8	5.5	5.4	5.1
1998	9.7	14.4	5.5	5.2	4.8	4.7	4.4	4.3	4.1
Average	17.8	12.2	7.1	6.8	6.5	6.3	6.1	6.0	5.6

Panel B. Average year-to-year earnings growth rate in given interval (%)											
	Interval of years, where 0 represents the origin of forecast										
Forecasts as of April	1	1-5	1-10	1-20	1-36	6-41	1-50	1-80	6-85	1-100	Asymp.
1985	16.8	12.9	10.8	9.7	9.2	8.6	9.0	8.9	8.6	8.8	8.4
1986	14.7	12.5	9.8	8.3	7.5	6.6	7.2	6.7	6.3	6.6	5.6
1987	27.4	15.8	11.5	9.3	8.1	6.8	7.7	7.1	6.5	6.9	5.6
1988	32.7	15.4	11.5	9.5	8.5	7.4	8.1	7.6	7.1	7.4	6.1
1989	14.1	11.2	9.7	8.8	8.3	7.8	8.0	7.7	7.5	7.6	6.4
1990	12.9	12.4	10.2	9.0	8.3	7.5	8.0	7.6	7.2	7.4	6.2
1991	10.5	13.6	10.4	8.7	7.8	6.9	7.5	7.0	6.5	6.8	5.5
1992	24.6	16.6	11.8	9.2	7.9	6.5	7.4	6.9	6.2	6.6	5.3
1993	19.3	15.2	10.6	8.2	7.0	5.7	6.5	6.0	5.4	5.8	4.8
1994	17.1	14.4	10.7	8.6	7.6	6.4	7.1	6.6	6.1	6.4	5.2
1995	21.8	14.9	11.1	9.0	7.9	6.7	7.4	6.9	6.3	6.7	5.5
1996	14.8	13.4	10.0	8.1	7.1	6.1	6.7	6.2	5.7	6.0	5.0
1997	12.3	13.5	10.2	8.3	7.4	6.3	7.0	6.5	6.0	6.2	5.1
1998	9.7	14.0	9.8	7.6	6.4	5.1	5.9	5.4	4.8	5.2	4.1
Average	17.8	14.0	10.6	8.7	7.8	6.7	7.4	6.9	6.4	6.8	5.6
Reference								CT: 6.6			

Table II b)

Forecast year-to-year book value growth rates

Panel A display marginal book value growth rates, b_t/b_{t-1}, where t is the number of years into the future relative to origin of forecast. b_0, b_1, ..., b_3, are taken from Table I. b_4, b_5,..., b_n are generated by (16). Panel B shows average growth rates in different intervals. PN refers to Penman and Nissim (2001) and the figures are the mean and, within parenthesis, the median.

Panel A. Marginal year-to-year book value growth rate (%)

	Year relative to origin of forecast (t)								
Forecasts as of April	1	5	10	20	36	50	80	100	Asymp.
1985	7.6	8.7	8.7	8.7	8.6	8.6	8.5	8.5	8.4
1986	7.3	8.5	8.0	7.4	6.8	6.5	6.1	6.0	5.6
1987	7.0	8.5	8.1	7.5	6.9	6.7	6.3	6.1	5.6
1988	7.8	8.7	8.3	7.9	7.4	7.2	6.8	6.7	6.1
1989	8.5	9.1	8.7	8.3	7.8	7.6	7.2	7.1	6.4
1990	7.9	9.1	8.7	8.1	7.6	7.3	7.0	6.8	6.2
1991	6.8	8.6	8.2	7.5	7.0	6.7	6.3	6.1	5.5
1992	6.6	8.5	8.0	7.3	6.7	6.3	5.9	5.8	5.3
1993	6.9	8.7	7.8	6.8	6.0	5.6	5.2	5.1	4.8
1994	7.8	9.4	8.5	7.5	6.6	6.3	5.8	5.6	5.2
1995	8.3	9.7	8.8	7.7	6.9	6.5	6.0	5.9	5.5
1996	8.1	9.4	8.4	7.3	6.4	6.0	5.5	5.3	5.0
1997	8.4	9.9	8.8	7.6	6.6	6.2	5.7	5.5	5.1
1998	8.5	10.4	8.6	6.9	5.7	5.2	4.7	4.5	4.1
Average	7.7	9.1	8.4	7.6	6.9	6.6	6.2	6.1	5.6

Panel B. Average year-to-year book value growth rate in interval (%)

	Interval of years, where 0 represents the origin of forecast										
Forecasts as of April	1	1-5	1-10	1-20	1-36	6-41	1-50	1-80	6-85	1-100	Asymp.
1985	7.6	8.2	8.5	8.6	8.6	8.7	8.6	8.6	8.6	8.6	8.4
1986	7.3	8.0	8.1	7.9	7.5	7.3	7.2	6.9	6.8	6.7	5.6
1987	7.0	7.9	8.1	7.9	7.6	7.4	7.4	7.0	6.9	6.9	5.6
1988	7.8	8.2	8.3	8.2	7.9	7.8	7.8	7.5	7.4	7.3	6.1
1989	8.5	8.7	8.8	8.6	8.4	8.2	8.2	7.9	7.8	7.7	6.4
1990	7.9	8.5	8.7	8.5	8.2	8.1	8.0	7.7	7.6	7.5	6.2
1991	6.8	7.9	8.1	8.0	7.6	7.5	7.4	7.0	6.9	6.9	5.5
1992	6.6	7.7	7.9	7.7	7.4	7.2	7.1	6.7	6.6	6.6	5.3
1993	6.9	8.0	8.0	7.6	7.0	6.8	6.7	6.2	6.0	6.0	4.8
1994	7.8	8.8	8.8	8.4	7.8	7.4	7.4	6.9	6.7	6.6	5.2
1995	8.3	9.1	9.1	8.6	8.0	7.7	7.6	7.1	6.9	6.9	5.5
1996	8.1	8.8	8.8	8.3	7.6	7.2	7.2	6.6	6.4	6.4	5.0
1997	8.4	9.2	9.2	8.6	7.9	7.5	7.5	6.9	6.7	6.6	5.1
1998	8.5	9.5	9.4	8.5	7.4	6.9	6.9	6.1	5.8	5.8	4.1
Average	7.7	8.5	8.6	8.2	7.8	7.6	7.5	7.1	6.9	6.9	5.6
Reference					PN: 11.3(9.0)						

Table II c)

Forecast return-on-equity

Panel A displays return-on-equity, e_t/b_{t-1}, where t is the number of years into the future relative to origin of forecast. b_0, b_1, ..., b_3, and e_0, e_1, ..., e_4, are taken from Table I. e_5, e_6, ..., e_n, are generated by equation (15), and b_4, b_5,..., b_n are generated by (16). Panel B displays average return-on-equity in different time intervals. PN refers to Penman and Nissim (2001), and the figures are the mean and, within parenthesis, the median.

Panel A. Marginal return-on-equity (%)

	Year relative to origin of forecast (t)								
Forecasts as of April	1	5	10	20	36	50	80	100	Asymp.
1985	15.2	17.4	17.4	17.3	17.2	17.2	17.1	17.0	16.86[#]
1986	14.7	17.0	16.0	14.7	13.6	12.9	12.2	12.0	11.27
1987	14.1	16.9	16.1	15.0	13.9	13.3	12.6	12.3	11.12
1988	15.6	17.3	16.7	15.8	14.9	14.3	13.7	13.4	12.15
1989	17.0	18.1	17.5	16.5	15.6	15.1	14.4	14.2	12.75
1990	15.7	18.1	17.4	16.3	15.2	14.6	13.9	13.6	12.33
1991	13.6	17.2	16.3	15.1	13.9	13.3	12.6	12.3	11.05
1992	13.2	17.0	15.9	14.5	13.3	12.7	11.9	11.6	10.57
1993	13.8	17.3	15.6	13.6	12.0	11.2	10.4	10.1	9.61
1994	15.7	18.8	17.1	15.0	13.3	12.5	11.6	11.3	10.48
1995	16.6	19.4	17.6	15.4	13.7	12.9	12.1	11.8	11.03
1996	16.1	18.9	16.9	14.5	12.7	11.9	11.0	10.7	9.96
1997	16.7	19.8	17.6	15.1	13.2	12.3	11.4	11.0	10.12
1998	16.9	20.8	17.2	13.8	11.4	10.4	9.3	9.0	8.15
Average	15.3	18.2	16.8	15.2	13.9	13.2	12.4	12.2	10.8

Panel B. Average return-on-equity in interval (%)

	Interval of years, where 0 represents the origin of forecast										
Forecasts as of April	1	1-5	1-10	1-20	1-36	6-41	1-50	1-80	6-85	1-100	Asymp.
1985	15.2	16.4	16.9	17.1	17.2	17.3	17.2	17.2	17.2	17.2	16.86[#]
1986	14.7	16.0	16.2	15.7	15.0	14.6	14.5	13.8	13.5	13.4	11.27
1987	14.1	15.8	16.1	15.8	15.1	14.9	14.7	14.0	13.8	13.7	11.12
1988	15.6	16.4	16.7	16.4	15.9	15.7	15.5	14.9	14.8	14.7	12.15
1989	17.0	17.4	17.6	17.3	16.7	16.4	16.3	15.7	15.5	15.4	12.75
1990	15.7	17.1	17.4	17.0	16.4	16.1	16.0	15.3	15.1	15.0	12.33
1991	13.6	15.8	16.2	15.9	15.3	15.0	14.8	14.1	13.9	13.8	11.05
1992	13.2	15.5	15.9	15.5	14.8	14.4	14.2	13.5	13.3	13.1	10.57
1993	13.8	15.9	16.1	15.2	14.1	13.5	13.4	12.4	12.0	12.0	9.61
1994	15.7	17.5	17.6	16.7	15.5	14.9	14.8	13.7	13.3	13.3	10.48
1995	16.6	18.3	18.2	17.3	16.0	15.4	15.2	14.2	13.8	13.7	11.03
1996	16.1	17.6	17.6	16.5	15.2	14.5	14.4	13.2	12.8	12.8	9.96
1997	16.7	18.4	18.4	17.2	15.8	15.0	14.9	13.7	13.3	13.2	10.12
1998	16.9	19.1	18.8	16.9	14.9	13.8	13.7	12.2	11.6	11.6	8.15
Average	15.3	16.9	17.1	16.5	15.6	15.1	15.0	14.1	13.9	13.8	10.8
Reference					PN: 10.2(12.2)						

Table II d)

Forecast *P/E* ratio

Panel A shows the *P/E* ratio, v_t/e_t, where t is the number of years into the future relative to origin of forecast. v_t is given by (10). e_0, e_1, ..., e_4, are taken from Table I,, and e_5, e_6, ..., e_n, are generated by equation (15). Panel B exhibits average *P/E* ratios in different time intervals.

Panel A. *P/E* ratio

	Year relative to origin of forecast (t)								
Forecasts as of April	1	5	10	20	36	50	80	100	Asymp.
1985	10.54	9.43	9.41	9.37	9.32	9.29	9.23	9.20	9.11
1986	13.78	11.43	11.18	10.80	10.39	10.16	9.84	9.72	9.37
1987	15.25	12.32	12.04	11.61	11.14	10.85	10.45	10.27	9.49
1988	12.69	11.17	10.96	10.61	10.22	9.98	9.65	9.50	8.73
1989	11.84	10.81	10.59	10.25	9.88	9.64	9.32	9.17	8.34
1990	13.21	11.19	10.95	10.56	10.14	9.89	9.53	9.38	8.61
1991	16.31	12.58	12.28	11.81	11.31	11.00	10.58	10.39	9.55
1992	17.05	12.86	12.55	12.07	11.54	11.23	10.81	10.63	9.96
1993	17.72	13.17	12.86	12.39	11.91	11.64	11.30	11.17	10.90
1994	16.68	12.90	12.53	11.98	11.41	11.09	10.67	10.51	10.05
1995	15.21	12.09	11.75	11.24	10.72	10.43	10.07	9.93	9.57
1996	17.10	13.38	12.99	12.43	11.85	11.52	11.11	10.94	10.54
1997	17.76	13.70	13.26	12.62	11.96	11.60	11.12	10.93	10.38
1998	23.68	16.63	16.12	15.36	14.57	14.13	13.56	13.34	12.77
Average	15.63	12.40	12.10	11.65	11.17	10.89	10.52	10.36	9.81

Panel B. Average *P/E* ratio in interval

	Interval of years, where 0 represents the origin of forecast										
Forecasts as of April	0-1	0-5	0-10	0-20	0-36	6-41	0-50	0-80	6-85	0-100	Asymp.
1985	10.91	10.17	9.83	9.62	9.50	9.36	9.45	9.38	9.31	9.34	9.11
1986	14.25	12.87	12.14	11.58	11.14	10.74	10.90	10.56	10.34	10.40	9.37
1987	16.65	14.28	13.31	12.59	12.05	11.54	11.75	11.34	11.06	11.14	9.49
1988	14.14	12.57	11.87	11.34	10.93	10.55	10.70	10.36	10.15	10.21	8.73
1989	12.16	11.58	11.17	10.80	10.47	10.20	10.27	9.97	9.81	9.83	8.34
1990	13.49	12.38	11.77	11.27	10.86	10.50	10.62	10.28	10.07	10.11	8.61
1991	16.52	14.50	13.54	12.81	12.26	11.74	11.95	11.51	11.22	11.30	9.55
1992	18.42	15.40	14.16	13.26	12.61	11.99	12.27	11.80	11.47	11.58	9.96
1993	18.78	15.87	14.56	13.62	12.97	12.33	12.64	12.19	11.86	12.00	10.90
1994	17.44	15.14	14.02	13.16	12.51	11.91	12.16	11.67	11.35	11.46	10.05
1995	16.22	14.10	13.09	12.31	11.72	11.17	11.40	10.97	10.67	10.77	9.57
1996	17.73	15.68	14.52	13.64	12.97	12.35	12.61	12.12	11.79	11.90	10.54
1997	18.20	16.13	14.90	13.94	13.21	12.53	12.81	12.26	11.90	12.01	10.38
1998	24.11	20.63	18.67	17.24	16.24	15.26	15.71	15.00	14.49	14.69	12.77
Average	16.36	14.38	13.40	12.66	12.10	11.58	11.80	11.39	11.11	11.20	9.81

Table II e)

Forecast *P/B* ratio

Panel A shows the *P/B* ratio, v_t/b_t, where *t* is the number of years into the future relative to origin of forecast. v_t is given by (10). b_0, b_1, ..., b_3, are taken from Table I, and b_4, b_5,..., b_n are generated by (16). Panel B exhibits average *P/B* ratios in different time intervals.

Panel A. *P/B* ratio

	Year relative to origin of forecast (*t*)									
Forecasts as of April	1	5	10	20	36		50	80	100	Asymp
1985	1.49	1.51	1.51	1.50	1.48		1.47	1.45	1.44	1.42
1986	1.88	1.79	1.66	1.48	1.32		1.24	1.13	1.10	1.00
1987	2.01	1.93	1.80	1.62	1.45		1.35	1.24	1.19	1.00
1988	1.83	1.78	1.69	1.55	1.41		1.34	1.23	1.19	1.00
1989	1.85	1.79	1.70	1.57	1.43		1.36	1.25	1.21	1.00
1990	1.93	1.86	1.75	1.59	1.43		1.35	1.24	1.20	1.00
1991	2.08	2.00	1.85	1.66	1.47		1.38	1.25	1.20	1.00
1992	2.11	2.01	1.85	1.64	1.44		1.34	1.21	1.16	1.00
1993	2.29	2.10	1.86	1.58	1.35		1.24	1.12	1.08	1.00
1994	2.42	2.22	1.97	1.67	1.42		1.31	1.17	1.12	1.00
1995	2.34	2.14	1.90	1.61	1.38		1.27	1.15	1.10	1.00
1996	2.55	2.31	2.02	1.68	1.42		1.30	1.16	1.11	1.00
1997	2.74	2.47	2.14	1.77	1.48		1.35	1.20	1.14	1.00
1998	3.69	3.13	2.56	1.98	1.57		1.39	1.21	1.14	1.00
Average	2.23	2.07	1.88	1.63	1.43		1.33	1.22	1.17	1.03

Panel B. Average *P/B* ratio in interval

	Interval of years, where 0 represents the origin of forecast										
Forecasts as of April	0-1	0-5	0-10	0-20	0-36	6-41	0-50	0-80	6-85	0-100	Asymp
1985	1.48	1.50	1.50	1.50	1.50	1.49	1.49	1.48	1.48	1.47	1.42
1986	1.88	1.85	1.79	1.68	1.55	1.47	1.47	1.36	1.31	1.31	1.00
1987	2.00	1.98	1.92	1.81	1.68	1.60	1.60	1.49	1.43	1.43	1.00
1988	1.83	1.81	1.77	1.69	1.60	1.53	1.54	1.44	1.40	1.39	1.00
1989	1.85	1.83	1.79	1.71	1.61	1.55	1.55	1.46	1.42	1.41	1.00
1990	1.92	1.90	1.85	1.76	1.65	1.57	1.57	1.47	1.42	1.42	1.00
1991	2.07	2.05	1.98	1.87	1.73	1.64	1.64	1.52	1.46	1.46	1.00
1992	2.10	2.07	2.00	1.87	1.72	1.62	1.63	1.49	1.43	1.43	1.00
1993	2.29	2.22	2.10	1.90	1.70	1.56	1.59	1.43	1.35	1.37	1.00
1994	2.43	2.35	2.22	2.01	1.80	1.65	1.68	1.51	1.43	1.44	1.00
1995	2.35	2.26	2.14	1.94	1.74	1.60	1.62	1.46	1.38	1.40	1.00
1996	2.56	2.46	2.31	2.07	1.84	1.67	1.70	1.52	1.43	1.44	1.00
1997	2.76	2.64	2.47	2.21	1.95	1.76	1.80	1.60	1.49	1.51	1.00
1998	3.74	3.49	3.16	2.70	2.28	1.98	2.06	1.77	1.61	1.65	1.00
Average	2.23	2.17	2.07	1.91	1.74	1.62	1.64	1.50	1.43	1.44	1.03

Table III

Relationships between discount rates, equity premia, asympotic growth rates of earnings, dividends, book values, key ratios and abnormal earnings growth

Panel A *contains* key quantities for the relationships between the discount rate (*r*), equity premium (*r-f*) and perpetual growth rate of abnormal earnings (*g*). *rgmax* is the upper bound imposed on *r* and *g* by equation (22) (or (23)). f is the 10-year risk-free rate. *rgmax-f* is the maximum equity premium. *gmin* is the *g* that corresponds to a equity premium (*r-f*) of zero. *rmin* is the asympote along which *g* viewed as a function of *r* approaches negative infinity when *r* approaches *rmin* from above. *gbal and rbal* solves *g=r – pr*, i.e., the “balance point”[#] for, *asympg*, the converging growth rate in earnings, dividends and book values. *mindrdg* is the derivative *dr/dg* evaluated at *gmin*. *maxdr/dg* is *dr/dg* evaluated at *rgmax*. *aROE* equals *r* when *r-pr>g*, and equals *g*/(1-*p*) when *r-pr<g*. *aP/B* and *aP/E* approaches infinity as *g* and *r* approaches *rgmax*. *aP/Bbal* corresponding to point of equivalence is 1. *aP/Egmin* corresponding to zero risk premium is given by 1-*p*+1/*f*. *aROErgmax* =*rgmax*/(1-*p*). *aROEbal*=*rbal*=*gbal*/(1-*p*). *aROEgmin*=*f*. *aROErmin*=*rmin*. *aP/Ebal* =1-*p*+1/*rbal*. *aP/Egmin* =1-*p*+1/*r*=1-*p*+1/*f*. *aP/Ermin* =1-*p*+1/*rmin*.

Panel B *replicates* some of CT’s results. *g=f – 3%* is CT’s forecasted growth in abnormal earnings and equal to the 10 year risk-free rate less 3%. *r* is the corresponding discount rate, and *r-f* the corresponding equity premium. *g>r – p r* is TRUE if *g* is larger than *r – p r*, and then *asympg*, the asymptotic growth rates of earnings, dividends and book values, equals *g*. If *g>r – p r* is FALSE then *asympg* is equal to *r – pr*.

	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	Mean
							Panel A								
rgmax (%)	17.4	17.0	16.9	17.3	18.1	18.1	17.2	17.0	17.3	18.8	19.4	18.9	19.8	20.8	18.2
f (%)	11.4	7.3	8.0	8.7	9.2	8.8	8.0	7.5	6.0	7.0	7.1	6.5	6.9	5.6	7.7
rgmax - f (%)	6.0	9.7	8.9	8.6	8.9	9.3	9.2	9.5	11.4	11.9	12.3	12.4	12.9	15.1	10.4
gmin (%)	-6.0	-10.3	-4.3	-6.4	-6.4	-5.9	-3.5	-4.2	-7.2	-5.1	-7.1	-5.0	-3.5	-2.0	-5.5
rmin (%)	4.6	-2.6	-4.2	-1.5	-1.0	-2.3	-5.1	-5.8	-7.7	-8.0	-6.7	-9.4	-10.6	-17.3	-5.5
gbal (%)	7.0	5.9	5.7	6.2	6.4	6.3	5.7	5.5	5.3	5.6	5.9	5.4	5.5	4.7	5.8
rbal (%)	14.0	11.9	11.4	12.3	12.8	12.5	11.3	11.0	10.7	11.2	11.9	10.9	10.9	9.3	11.6
mindrdg	0.15	0.21	0.28	0.22	0.22	0.24	0.30	0.30	0.30	0.33	0.29	0.35	0.39	0.51	0.29
maxdrdg	0.41	0.56	0.59	0.55	0.55	0.58	0.61	0.62	0.66	0.68	0.67	0.70	0.73	0.81	0.62
aROErgmax (%)	34.9	34.1	33.9	34.6	36.2	36.3	34.5	33.9	34.7	37.7	38.8	37.8	39.6	41.6	36.3
aROEbal (%)	14.0	11.9	11.4	12.3	12.8	12.5	11.3	11.0	10.7	11.2	11.9	10.9	10.9	9.3	11.6
aROEgmin (%)	11.4	7.3	8.0	8.7	9.2	8.8	8.0	7.5	6.0	7.0	7.1	6.5	6.9	5.6	7.7
aROErmin (%)	4.6	-2.6	-4.2	-1.5	-1.0	-2.3	-5.1	-5.8	-7.7	-8.0	-6.7	-9.4	-10.6	-17.3	-5.5
aP/Bbal	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
aP/Ebal	7.6	8.9	9.3	8.6	8.3	8.5	9.3	9.6	9.9	9.4	8.9	9.7	9.7	11.2	9.2
aP/Egmin	9.2	14.2	13.0	12.0	11.4	11.9	12.9	13.9	17.3	14.8	14.7	15.9	15.0	18.2	13.9
aP/Ermin	22.4														
							Panel B. CT’s data								
g= f - 3% (%)	8.4	4.3	5.0	5.7	6.2	5.8	5.0	4.5	3.0	4.0	4.1	3.5	3.9	2.6	4.7
r (%)	14.4	11.3	11.1	12.2	12.8	12.3	11.1	10.6	9.6	10.5	11.0	10.0	10.1	8.2	11.1
r-f (%)	3.0	4.0	3.1	3.4	3.6	3.5	3.0	3.1	3.7	3.5	4.0	3.5	3.2	2.5	3.4
g>r-pr	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	
asympg (%)	8.4	5.6	5.6	6.1	6.4	6.2	5.5	5.3	4.8	5.2	5.5	5.0	5.1	4.1	5.6
aROE (%)	16.9	11.3	11.1	12.2	12.8	12.3	11.1	10.6	9.6	10.5	11.0	10.0	10.1	8.2	11.2
aP/B	1.42	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.03
aP/E	9.1	9.4	9.5	8.7	8.3	8.6	9.5	10.0	10.9	10.1	9.6	10.5	10.4	12.8	9.8

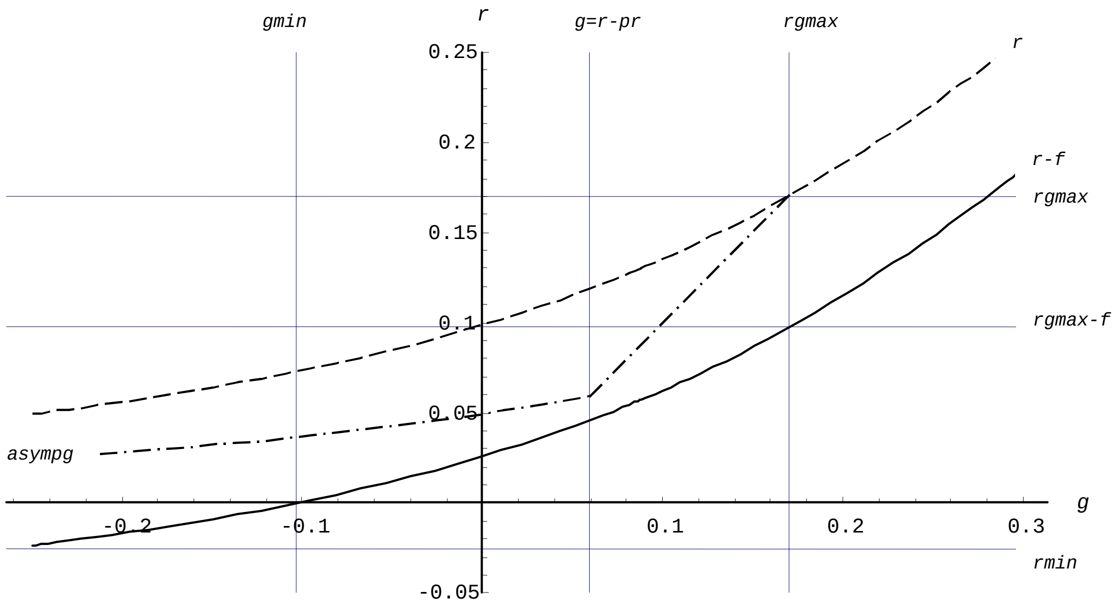

Figure 1. Relationships for 1986. The relationship defined by (18) between *r* and *g* for 1986 is represented by the dashed curve, with *r* displayed on the vertical axis and *g* on the horizontal. The risk premium, equal to *r-f*, plots as a solid curve and is labeled *r-f*. The valuation equation (5) was derived under the assumption that r>g, therefore *r* and *g* will have a common limiting upper value *rgmax*, i.e., *r*, *g*<*rgmax*. *rgmax* is found by setting g equal to *r* and solve for *r* in (16). *rgmax* is represented by two thin lines, one vertical and one horizontal, marking the upper bounds for *r* and *g*, respectively. The upper bound on the risk premium, *rgmax-f*, plots as a thin horizontal line. Graphed as a thin vertical line is *gmin*, which is the value of *g* that corresponds to a risk premium equal to zero. For reference, *rmin*, an even lower bound on *r* than the one posed by the risk-free rate, is calculated by setting the denominator of (18) equal to zero and solving for *r*. *rmin* is the largest positive real root to that equation and plots as a thin horizontal line. The asymptotic growth rate of earnings, dividends and book values, *asympg*, is equal to Max(*g, r – pr*), and displayed as a dash-dot curve. The "balance point", where *g* = *r* – *pr*, is marked by a thin solid vertical line crossing the horizontal axis where *g* = *r* – *pr*. To the right of this line, for larger values of *g*, the asymptotic growth rate for earnings, dividends and book values is equal to *g*.

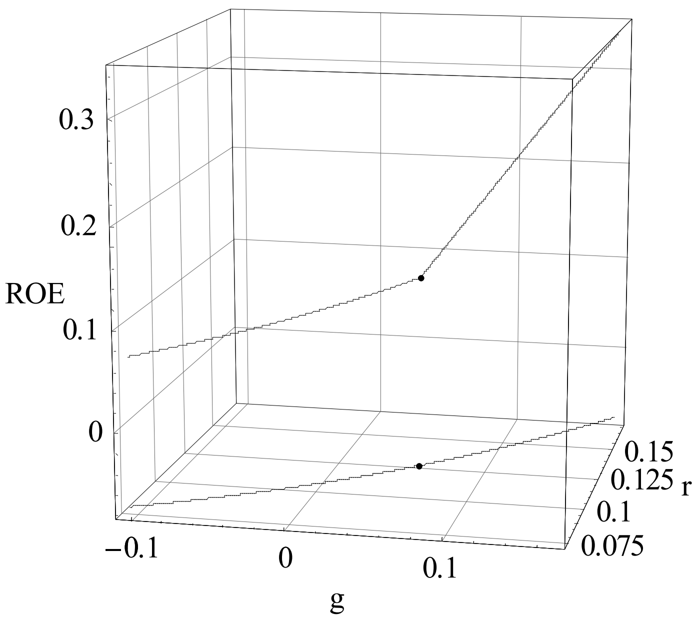

Figure 2. Relationship for asymptotic *ROE*, *r* and *g* for 1986.

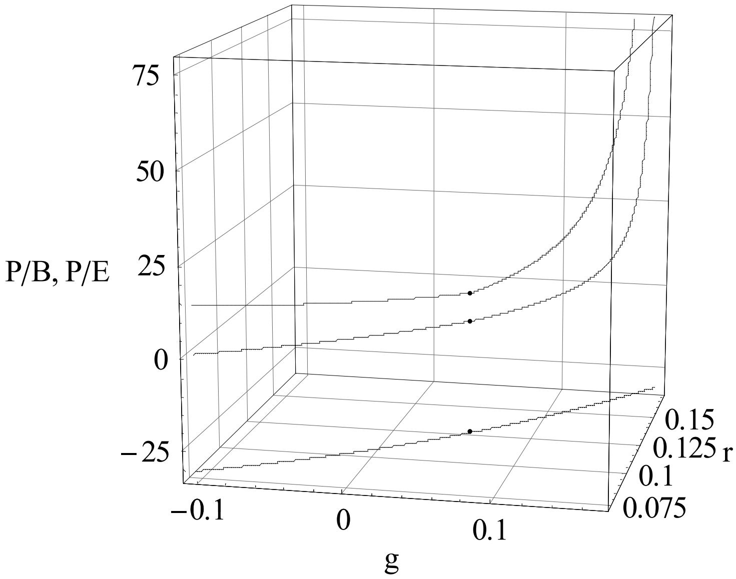

Figure 3. Relationships for asymptotic *P/E*, asymptotic *P/B, r* and *g* for 1986.