

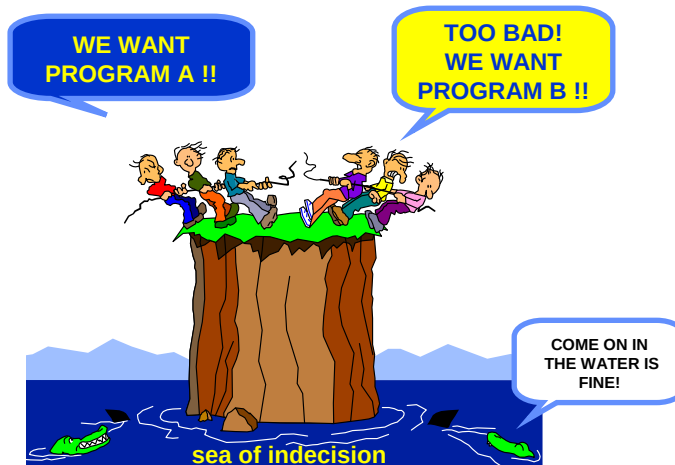
An Illustrated Guide to the *ANALYTIC HIERARCHY PROCESS*

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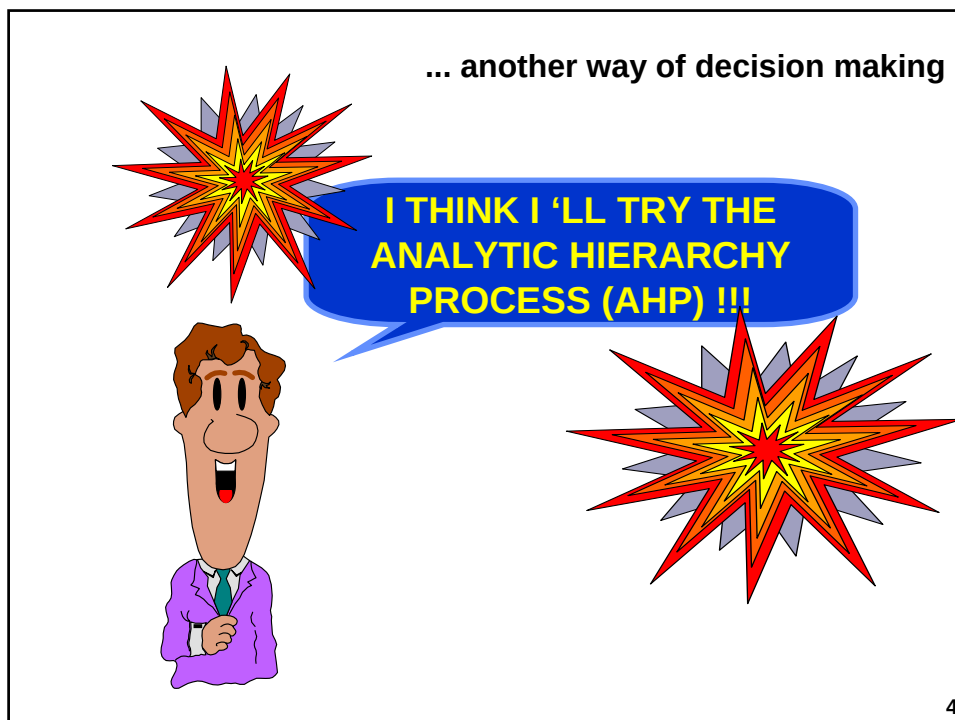
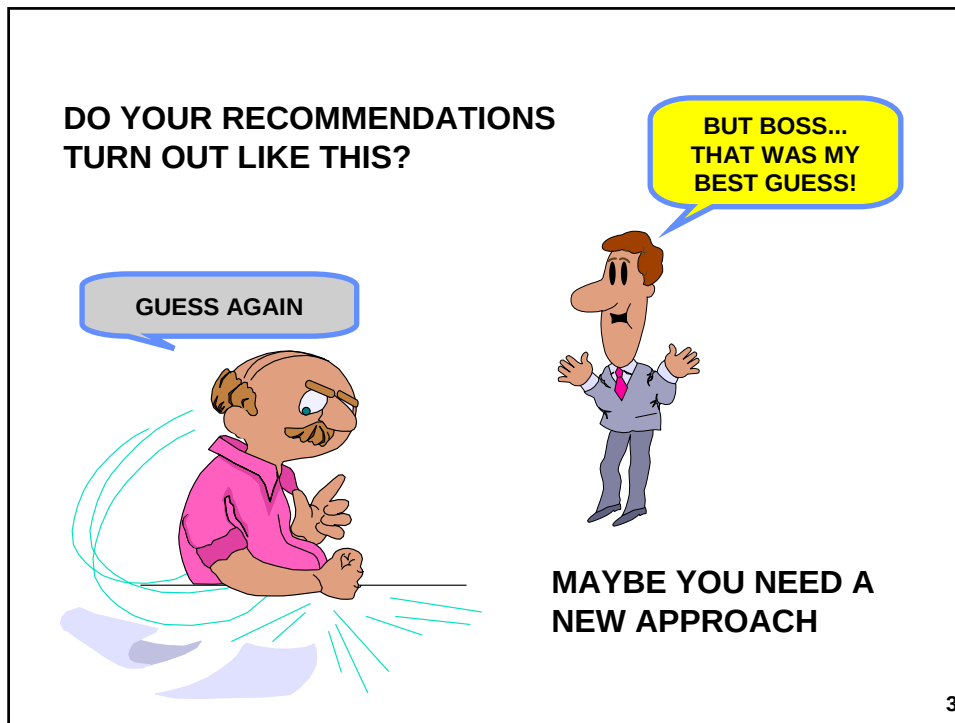
1

Do your decision conferences turn out like this?

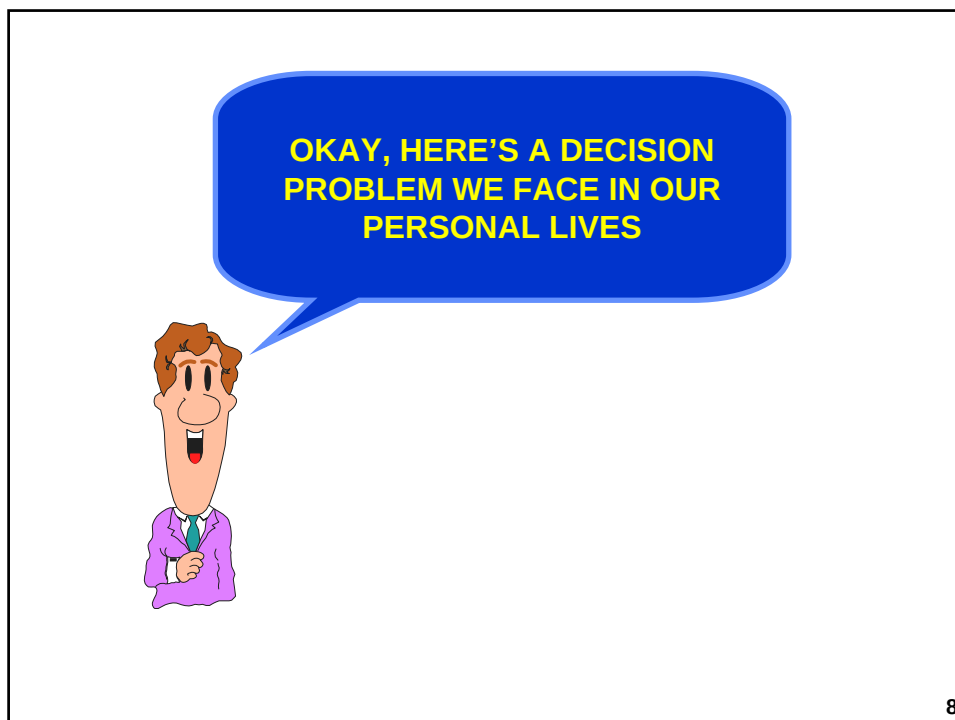
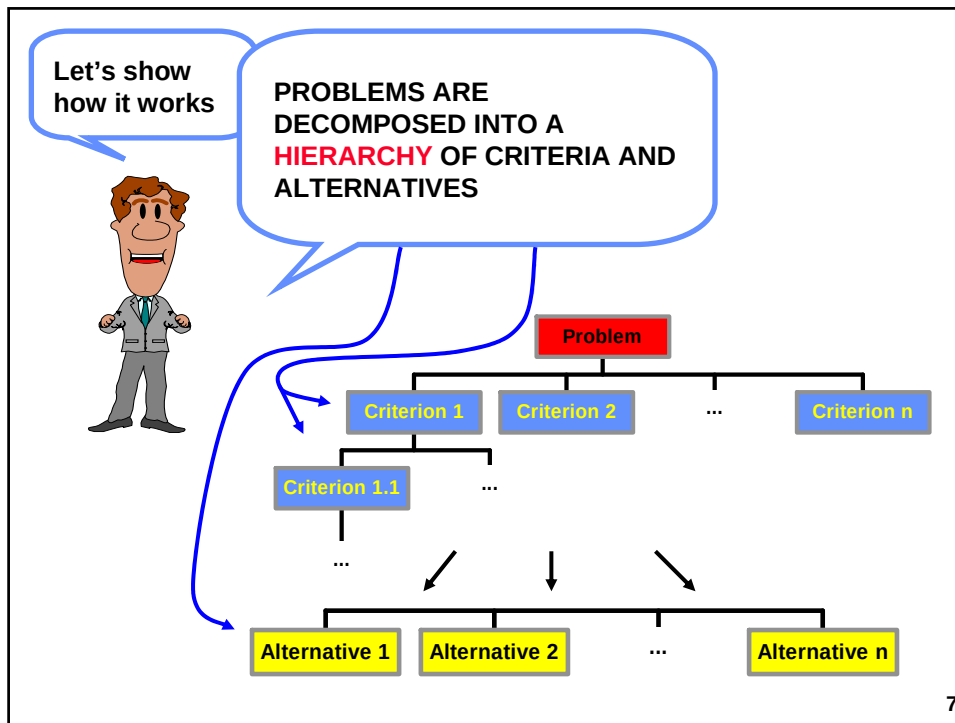


or does this happen?

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AN IMPORTANT PART OF THE
PROCESS IS TO ACCOMPLISH THESE
THREE STEPS

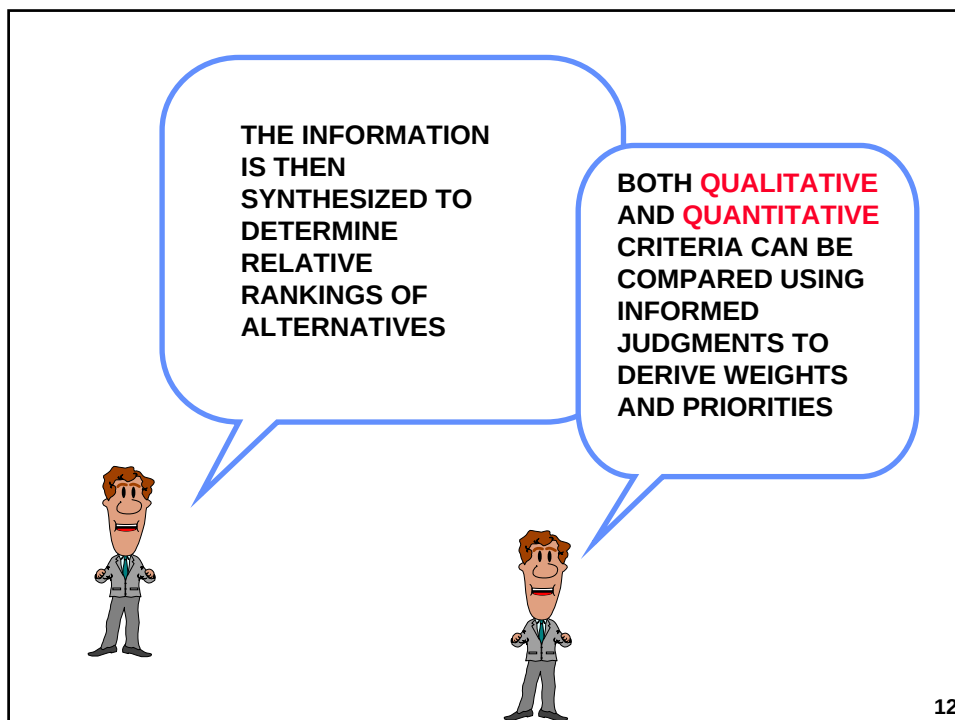
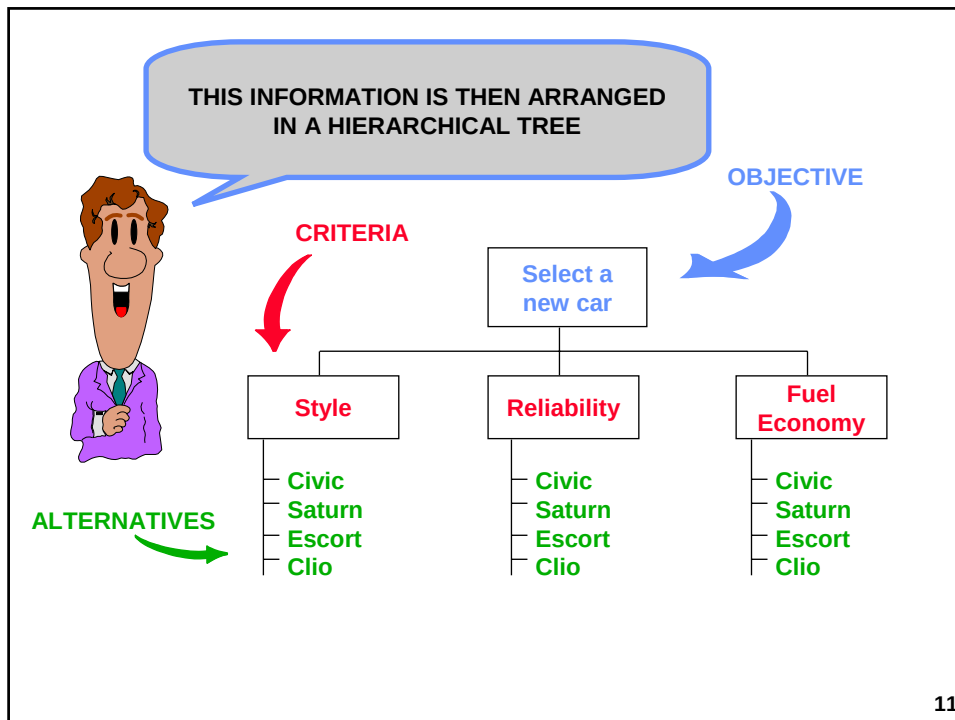
- STATE THE OBJECTIVE:
 - SELECT A NEW CAR
- DEFINE THE CRITERIA:
 - STYLE, RELIABILITY, FUEL ECONOMY
- PICK THE ALTERNATIVES:
 - CIVIC COUPE, SATURN COUPE, FORD ESCORT,
RENAULT CLIO



WHAT ABOUT COST?

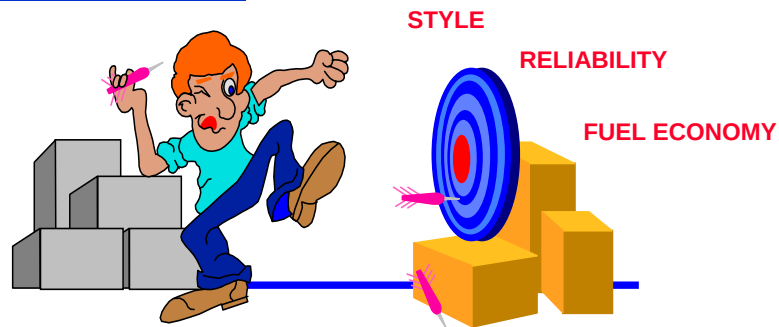
(BE QUIET, WE'LL TALK ABOUT THAT LATER)

SKEPTIC-GATOR



HOW DO YOU DETERMINE THE RELATIVE IMPORTANCE OF THE CRITERIA?

Here's one way !

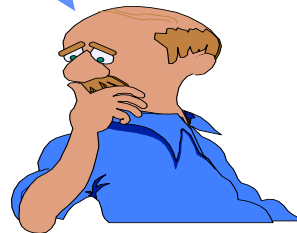


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HERE'S ANOTHER WAY

Hmm, I think reliability is the most important followed by style and fuel economy is least important so I will make the following judgements

USING **JUDGMENTS** TO DETERMINE THE RANKING OF THE CRITERIA



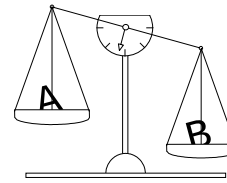
1. **RELIABILITY** IS 2 TIMES AS IMPORTANT AS **STYLE**
2. **STYLE** IS 3 TIMES AS IMPORTANT AS **FUEL ECONOMY**
3. **RELIABILITY** IS 4 TIMES AS IMPORTANT AS **FUEL ECONOMY**

he's not very consistent here ... that's o.k.

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Pairwise Comparisons

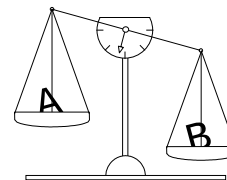


USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED

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Pairwise Comparisons



USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED

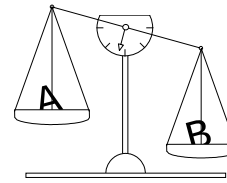
1 equal 3 moderate 5 strong 7 very strong 9 extreme

	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY		1/1	4/1
FUEL ECONOMY			1/1

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Pairwise Comparisons



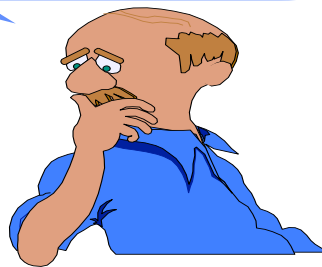
USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED

1 equal 3 moderate 5 strong 7 very strong 9 extreme

	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

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How do you turn this MATRIX into ranking of criteria?

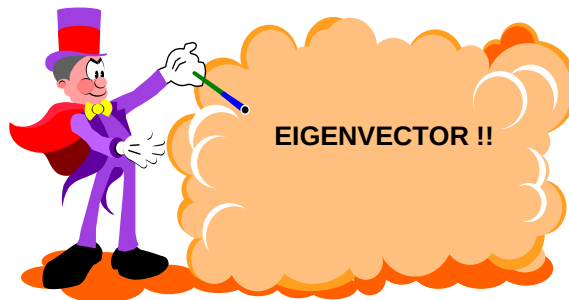


	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

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HOW DO YOU GET A RANKING OF PRIORITIES FROM A PAIRWISE MATRIX?

AND THE SURVEY SAYS



ACTUALLY...

DR THOMAS L. SAATY, CURRENTLY WITH THE UNIVERSITY OF PITTSBURGH, DEMONSTRATED MATHEMATICALLY THAT THE EIGENVECTOR SOLUTION WAS THE BEST APPROACH.

REFERENCE : THE ANALYTIC HIERARCHY PROCESS, 1990, THOMAS L. SAATY

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HERE'S HOW TO SOLVE FOR THE EIGENVECTOR:

1. A SHORT COMPUTATIONAL WAY TO OBTAIN THIS RANKING IS TO RAISE THE PAIRWISE MATRIX TO POWERS THAT ARE SUCCESSIVELY SQUARED EACH TIME.
2. THE ROW SUMS ARE THEN CALCULATED AND NORMALIZED.
3. THE COMPUTER IS INSTRUCTED TO STOP WHEN THE DIFFERENCE BETWEEN THESE SUMS IN TWO CONSECUTIVE CALCULATIONS IS SMALLER THAN A PRESCRIBED VALUE.

SAY WHAT!

SHOW ME AN EXAMPLE



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IT'S MATRIX ALGEBRA TIME !!!



	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

FOR NOW, LET'S REMOVE THE NAMES AND CONVERT THE FRACTIONS TO DECIMALS :

1.0000	0.5000	3.0000
2.0000	1.0000	4.0000
0.3333	0.2500	1.0000

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STEP 1: SQUARING THE MATRIX

THIS TIMES

1.0000	0.5000	3.0000
2.0000	1.0000	4.0000
0.3333	0.2500	1.0000

THIS

1.0000	0.5000	3.0000
2.0000	1.0000	4.0000
0.3333	0.2500	1.0000

I.E. $(1.0000 * 1.0000) + (0.5000 * 2.0000) + (3.0000 * 0.3333) = 3.0000$

RESULTS IN THIS

3.0000	1.7500	8.0000
5.3332	3.0000	14.0000
1.1666	0.6667	3.0000

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STEP 2 : NOW, LET'S COMPUTE OUR FIRST EIGENVECTOR
(TO FOUR DECIMAL PLACES)

FIRST, WE SUM THE ROWS

3.0000	+	1.7500	+	8.0000	=	12.7500	0.3194
5.3332	+	3.0000	+	14.0000	=	22.3332	0.5595
1.1666	+	0.6667	+	3.0000	=	4.8333	0.1211

SECOND, WE SUM THE ROW TOTALS → 39.9165 1.0000

FINALLY, WE **NORMALIZE** BY DIVIDING
THE ROW SUM BY THE ROW TOTALS
(I.E. 12.7500 DIVIDED BY 39.9165 EQUALS 0.3194)

THE RESULT IS OUR EIGENVECTOR
(A LATER SLIDE WILL EXPLAIN THE
MEANING IN TERMS OF OUR EXAMPLE)

0.3194
0.5595
0.1211

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THIS PROCESS MUST BE ITERATED UNTIL THE EIGENVECTOR
SOLUTION DOES NOT CHANGE FROM THE PREVIOUS ITERATION
(REMEMBER TO FOUR DECIMAL PLACES IN OUR EXAMPLE)

CONTINUING OUR EXAMPLE,
AGAIN, STEP 1: WE SQUARE THIS MATRIX

3.0000	1.7500	8.0000
5.3332	3.0000	14.0000
1.1666	0.6667	3.0000

WITH THIS RESULT →

27.6653	15.8330	72.4984
48.3311	27.6662	126.6642
10.5547	6.0414	27.6653

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AGAIN STEP 2 : COMPUTE THE EIGENVECTOR (TO FOUR DECIMAL PLACES)

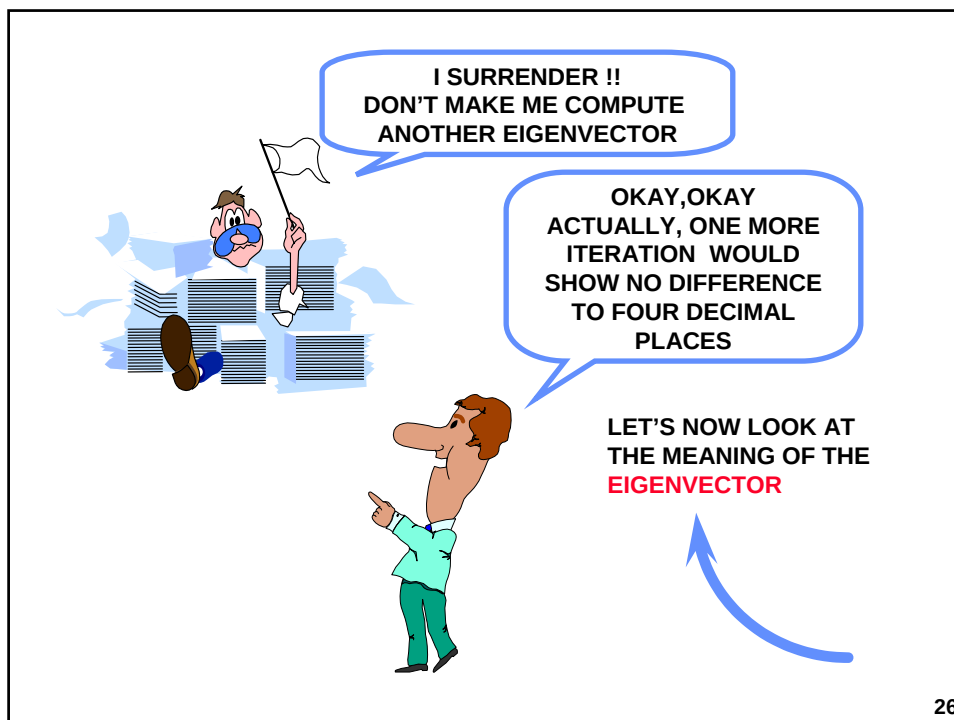
27.6653	+	15.8330	+	72.4984	=	115.9967	0.3196
48.3311	+	27.6662	+	126.6642	=	202.6615	0.5584
10.5547	+	6.0414	+	27.6653	=	44.2614	0.1220
					TOTALS	362.9196	1.0000

COMPUTE THE DIFFERENCE OF THE
PREVIOUS COMPUTED EIGENVECTOR
TO THIS ONE:

0.3194	—	0.3196	=	- 0.0002
0.5595	—	0.5584	=	0.0011
0.1211	—	0.1220	=	- 0.0009

TO FOUR DECIMAL PLACES THERE'S NOT MUCH DIFFERENCE
HOW ABOUT ONE MORE ITERATION?

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HERE'S OUR PAIRWISE MATRIX WITH THE NAMES



	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

AND THE COMPUTED EIGENVECTOR GIVES US THE RELATIVE RANKING OF OUR CRITERIA

STYLE	0.3196	← THE SECOND MOST IMPORTANT CRITERION
RELIABILITY	0.5584	← THE MOST IMPORTANT CRITERION
FUEL ECONOMY	0.1220	← THE LEAST IMPORTANT CRITERION

NOW BACK TO THE HIEARCHICAL TREE...

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HERE'S THE TREE WITH THE CRITERIA WEIGHTS



OBJECTIVE

CRITERIA

Style .3196

Reliability .5584

Fuel Economy .1220

ALTERNATIVES

Civic
Saturn
Escort
Clio

Civic
Saturn
Escort
Clio

Civic
Saturn
Escort
Clio

WHAT ABOUT THE ALTERNATIVES?

I'M GLAD YOU ASKED...

SKEPTIC-GATOR

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IN TERMS OF STYLE, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER



STYLE

	CIVIC	SATURN	ESCORT	CLIO
CIVIC	1/1	1/4	4/1	1/6
SATURN	4/1	1/1	4/1	1/4
ESCORT	1/4	1/4	1/1	1/5
CLIO	6/1	4/1	5/1	1/1

AND...

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IN TERMS OF RELIABILITY, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER



RELIABILITY

	CIVIC	SATURN	ESCORT	CLIO
CIVIC	1/1	2/1	5/1	1/1
SATURN	1/2	1/1	3/1	2/1
ESCORT	1/5	1/3	1/1	1/4
CLIO	1/1	1/2	4/1	1/1

ITS MATRIX ALGEBRA TIME!!!

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COMPUTING THE EIGENVECTOR
DETERMINES THE RELATIVE
RANKING OF ALTERNATIVES
UNDER EACH CRITERION



RANKING

STYLE

RANKING

RELIABILITY

3	CIVIC	.1160
2	SATURN	.2470
4	ESCORT	.0600
1	CLIO	.5770

1	CIVIC	.3790
2	SATURN	.2900
4	ESCORT	.0740
3	CLIO	.2570

WHAT ABOUT FUEL ECONOMY?



SKEPTIC-GATOR

ANOTHER GOOD QUESTION...

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AS STATED EARLIER,
AHP CAN COMBINE BOTH QUALITATIVE
AND QUANTITATIVE INFORMATION



FUEL ECONOMY INFORMATION IS OBTAINED FOR EACH
ALTERNATIVE:

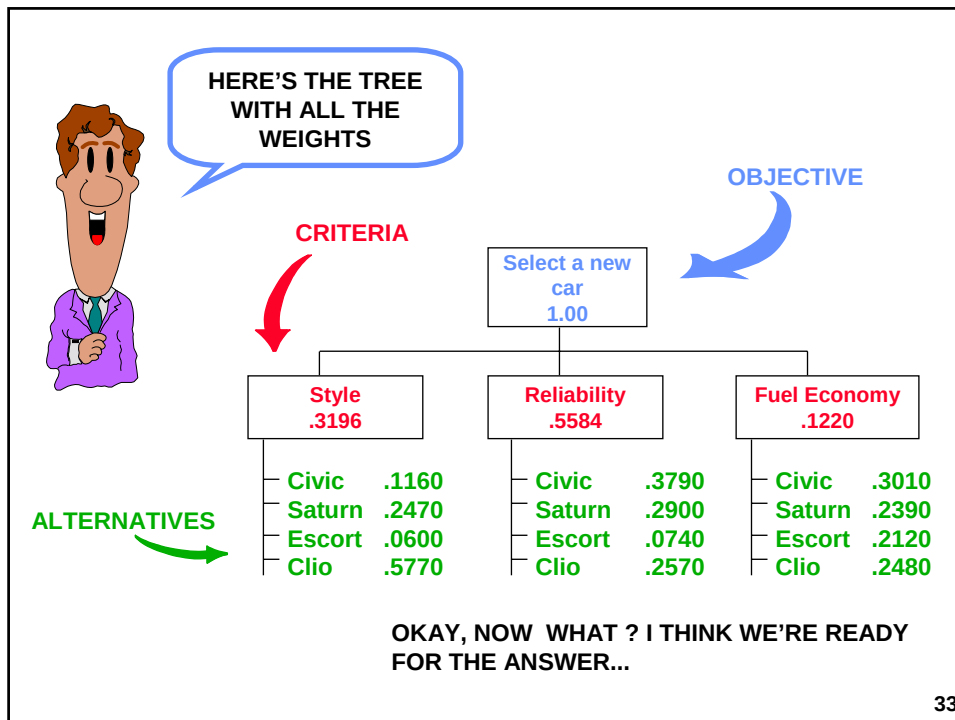
FUEL ECONOMY
(MILES/GALLON)

CIVIC	34	$34 / 113 =$	.3010
SATURN	27	$27 / 113 =$	.2390
ESCORT	24	$24 / 113 =$	.2120
CLIO	28	$28 / 113 =$	.2480
	113		1.0000

NORMALIZING THE FUEL ECONOMY INFO
ALLOWS US TO USE IT WITH OTHER RANKINGS



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A LITTLE MORE MATRIX ALGEBRA GIVES US THE SOLUTION:

	STYLE	RELIABILITY	FUEL ECONOMY	CRITERIA RANKING	
CIVIC	.1160	.3790	.3010	0.3196	STYLE
SATURN	.2470	.2900	.2390	0.5584	RELIABILITY
ESCORT	.0600	.0740	.2120	0.1220	FUEL ECONOMY
CLIO	.5770	.2570	.2480		

I.E. FOR THE CIVIC $(.1160 * .3196) + (.3790 * .5584) + (.3010 * .1220) = .3060$

Civic	.3060
Saturn	.2720
Escort	.0940
Clio	.3280

AND THE WINNER IS !!!

THE CLIO IS THE HIGHEST RANKED CAR

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IN SUMMARY, THE ANALYTIC HIERARCHY
PROCESS PROVIDES A LOGICAL FRAMEWORK
TO DETERMINE THE BENEFITS OF EACH
ALTERNATIVE



- | | |
|-----------|-------|
| 1. Clio | .3280 |
| 2. Civic | .3060 |
| 3. Saturn | .2720 |
| 4. Escort | .0940 |

WHAT ABOUT COSTS?



SKEPTIC-GATOR

WELL, I'LL TELL YOU...

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ALTHOUGH COSTS COULD HAVE BEEN
INCLUDED, IN MANY COMPLEX DECISIONS,
COSTS SHOULD BE SET ASIDE UNTIL THE
BENEFITS OF THE ALTERNATIVES ARE
EVALUATED



OTHERWISE THIS COULD HAPPEN...

YOUR PROGRAM COST TOO MUCH I
DON'T CARE ABOUT ITS BENEFITS



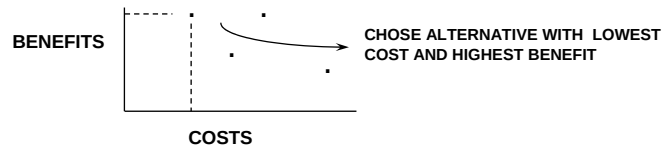
DISCUSSING COSTS
TOGETHER WITH BENEFITS
CAN SOMETIMES BRING FORTH
MANY POLITICAL AND
EMOTIONAL RESPONSES

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WAYS TO HANDLE BENEFITS AND COSTS INCLUDE THE FOLLOWING:



1. GRAPHING BENEFITS AND COSTS OF EACH ALTERNATIVE



2. BENEFIT TO COST RATIOS

3. LINEAR PROGRAMMING

4. SEPARATE BENEFIT AND COST HIERARCHICAL TREES AND THEN COMBINE THE RESULTS

IN OUR EXAMPLE...

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LET'S USE BENEFIT TO COST RATIOS



	COST \$	NORMALIZED COSTS	BENEFIT - COST RATIOS
1. CLIO	18,000	.3333	.3280 / .3333 = .9840
2. CIVIC	12,000	.2222	.3060 / .2222 = 1.3771
3. SATURN	15,000	.2778	.2720 / .2778 = .9791
4. ESCORT	9,000	.1667	.0940 / .1667 = .5639
	54,000	1.0000	

(REMEMBER THE BENEFITS WERE DERIVED EARLIER FROM THE AHP)

AND...

THE CIVIC IS THE WINNER WITH THE HIGHEST BENEFIT TO COST RATIO

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AHP CAN BE USED FOR VERY COMPLEX DECISIONS

MANY LEVELS OF CRITERIA AND SUBCRITERIA CAN BE INCLUDED

HERE'S SOME EXAMPLES

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AHP CAN BE USED FOR A WIDE VARIETY OF APPLICATIONS

STRATEGIC PLANNING

RESOURCE ALLOCATION

SOURCE SELECTION

BUSINESS/PUBLIC POLICY

PROGAM SELECTION

AND MUCH MUCH MORE...

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