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PERSPECTIVES

Fundamentally Flawed Indexing

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The capitalization-weighted equity market portfolio holds a special place in modern-day investing—and for good reason. The cap-weighted portfolio offers broad diversification and low transaction costs. Capitalization weighting is also the only strategy that all investors can follow. Because the collective holdings of investors (by definition) aggregate to the market portfolio, for every investor who is underweight a stock, another is overweight that stock, and between them, it is at best a zero-sum game. After fees and transaction costs, the average investor who deviates from capitalization weights must underperform the market portfolio.

Now, however, a new theory of finance is being advanced as providing definitive proof that holding stocks in proportion to their market capitalizations is an inferior investment strategy. The claim is that capitalization weighting necessarily invests more in overvalued stocks and less in undervalued stocks. Dubbed the "noisy market hypothesis," the theory is being used to advocate investments in non-cap-weighted (sometimes called "fundamental") funds and indices.[1]

Unfortunately, there is a fundamental flaw in the logic. As I will explain in detail, the "theory" is seeking to position an active management strategy in a passive management framework. And it *asserts* rather than derives the inferiority of capitalization weighting. The assertion, moreover, is false. Bottom line? Capitalization weighting does *not* impose an inherent performance drag.

The noisy market hypothesis goes like this: Start with the premise that the market errs in its pricing of individual stocks but that the pricing errors occur in a random fashion, so some stocks are overvalued while others are undervalued. Overvalued stocks have inflated market capitalizations and will have lower future returns; undervalued stocks have depressed market capitalizations and will have higher future returns. Accordingly, a cap-weighted strategy will systematically skew the portfolio toward investment in overvalued stocks, and such a strategy will not, therefore, perform as well as an approach that avoids the use of market capitalization in determining the individual stock weights.

Robert Arnott, one of the prime architects of fundamental indexing, has written:[2]

> No longer must investors suffer a performance drag by settling for an index that inherently overweights every overvalued company and underweights every undervalued one. With due respect to the pioneers in finance theory and the cap-weighted indexers, there is a better way. (Arnott 2006, p. 41)

Fundamental indexing proponent Jeremy Siegel has stated that "we are on the edge of a revolution" and has called the noisy market hypothesis a "new paradigm for understanding how markets work." He stated that it

> can be rigorously proved that if stock prices are subject to noise, then capitalization-weighted indexes will offer investors risk-and-return characteristics that are inferior to those of fundamentally weighted indexes. (Siegel 2006, p. A14)[3]

If valid, it would represent a profound finding that capitalization weighting, by itself, creates a mathematical headwind against performance—profound in that investors can benefit simply by avoiding capitalization weighting and profound in that skill in distinguishing overvalued from undervalued stocks is not required to obtain superior investment performance.

But the proposition is not valid. With due respect to proponents of fundamental indexing, I counter that it is not yet time to rewrite the finance text books. In what follows, I lay out what the noisy market hypothesis is claiming and then explain why the conclusion it reaches about the inferiority of capitalization weighting is incorrect.

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The Noisy Market Hypothesis and Its Fallacy

The noisy market hypothesis starts with the assumption that any given stock is as likely to be overvalued as undervalued. This statement may or may not be a good representation of real-world capital markets, but that is not the issue. The problem arises in going from this assumption about market prices to the conclusion that capitalization weighting systematically skews investment toward overvalued stocks. The issue is best illuminated by means of an example. A formal model is developed in Appendix A.

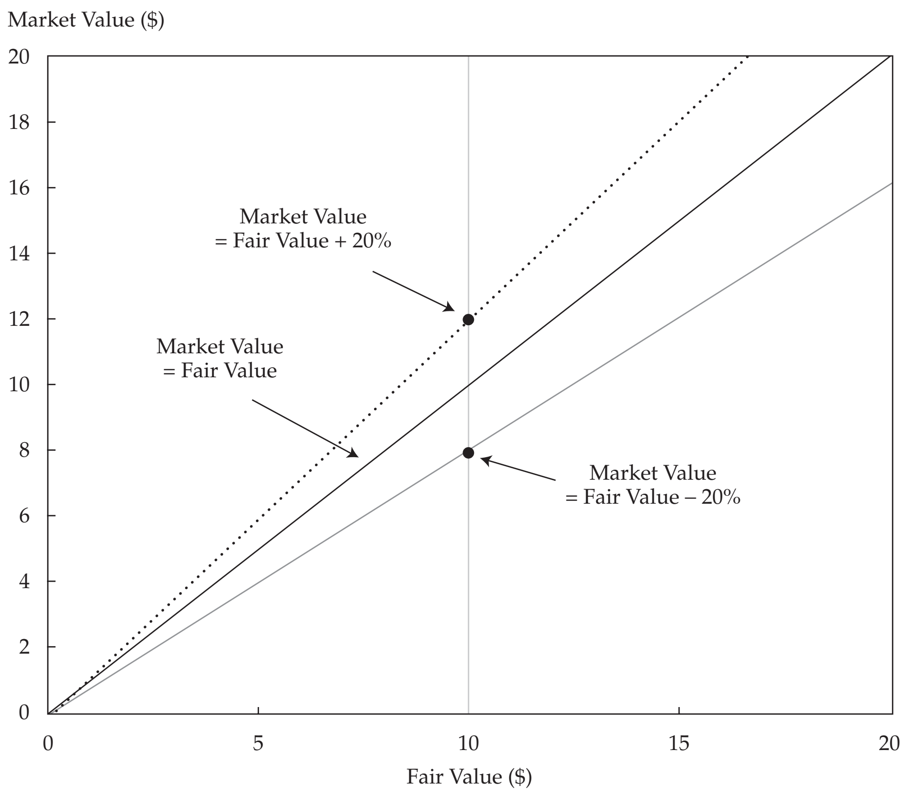
Consider forming a portfolio of the shares of just two companies, Company A and Company B. To keep things simple, suppose that the two companies are actually simply riskless closed-end money market funds. Both have net asset values of \$10 per share, and both yield the market rate of interest, which is 10 percent per year. In a year, these funds will each be liquidated and investors will receive \$11 per share risk free.

The market knows that Company A and Company B are closed-end money market funds, but it does not know their net asset values per share. In accordance with the noisy market hypothesis, assume the market misprices these shares in either direction and with equal probability. Suppose further that the mispricing is 20 percent of fair value. A and B thus will trade at either \$12 or \$8 per share (as illustrated in **Figure 1**). At \$12 per share, investors will lose 8.5 percent over the year because they will get back only \$11. At \$8 per share, investors will make 37.5 percent on their shares over the year.

Suppose that A and B have the same number of shares outstanding. Their market capitalizations thus will be in the proportion 60/40 (\$12/\$8) if A is overvalued and B is undervalued and will be in the proportion 40/60 if A is undervalued and B is overvalued. If both are overvalued or both are undervalued, the cap-weighted index will give equal weight to A and B. The cap-weighted index thus differs from the equally weighted portfolio when one stock is undervalued and the other is overvalued.

Continuing with this line of reasoning, the purported returns of the cap-weighted and equally weighted portfolios can now be calculated, as shown in **Table 1**. The analysis indicates that the cap-weighted index underperforms the equally weighted index—on average by 2.3 percent—

Figure 1. Market Value Given Fair Value



Note: Stock randomly over/underpriced by 20 percent.

Table 1. Cap-Weighted vs. Equally Weighted Returns: Perspective of the Noisy Market Hypothesis

Outcome	Cap Weights	Probability	Cap-Weighted Return	Equally Weighted Return
Both overvalued	50/50	25%	–8.3%	–8.3%
Both undervalued	50/50	25	37.5	37.5
One overvalued, one undervalued	60/40 or 40/60	50	10.0	14.6
Expected return			12.3	14.6

because capitalization weighting puts more weight on overvalued stocks than on undervalued stocks. This is the argument set forth by Arnott, Siegel, and others.

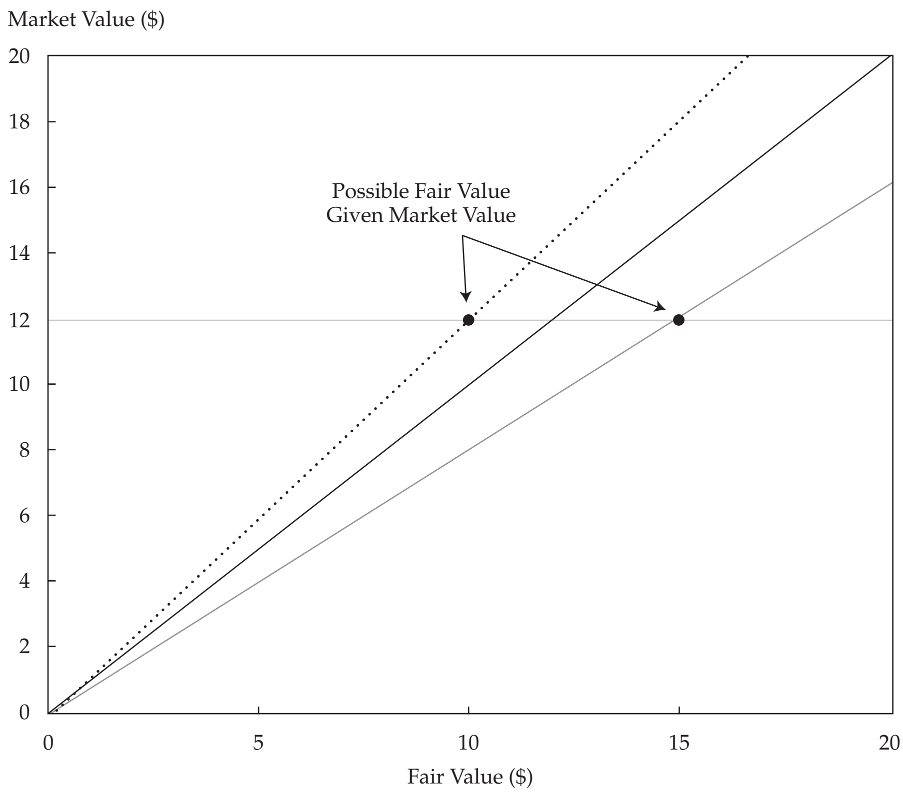
I will now explain where the problem lies.

Take the case in which Stock A is trading at $12 per share, and consider what investors actually know about this stock. They know its current share price is $12, and they know that A is either overvalued or undervalued by 20 percent with equal probability. They also know that A is a closed-end money market fund yielding 10 percent per year that will be liquidated in a year.

Crucially, investors do *not* know that the fair market value is $10/share. If they did know fair value was $10/share, the decision not to own any of A when it is trading at $12/share would be easy.

Given that the stock is trading at $12/share, there are only two possibilities: The fair market value of A must be either $10/share as a result of A being overvalued by 20 percent or $15 per share as a result of A being undervalued by 20 percent (as illustrated in **Figure 2**). If the fair value is $10, shareholders will receive $11 in one year, thus suffering a loss of 8.3 percent; if the fair value is $15, shareholders will receive $16.50 in one year, for a return of 37.5 percent. At $12 per share, and because undervaluation and overvaluation occur with equal probability, the expected return on the stock is 14.6 percent.

Figure 2. Fair Value Given Market Value



Note: Stock randomly over/underpriced by 20 percent.

A similar analysis applies at $8/share. Given this price level, there is a 50 percent chance that the stock is undervalued by 20 percent and thus truly worth $10/share today and $11/share in a year; and there is a 50 percent chance that it is overvalued by 20 percent and thus truly worth $6.67/share today and $7.33/share in a year. Once again, the return on the stock is either –8.3 percent or 37.5 percent, for an expected return of 14.6 percent.

Importantly, in these calculations, I am performing a Bayesian analysis: The noisy market hypothesis tells us that investors have an "uninformed prior distribution" on the fair value of the stock, meaning that before observing the price at which the shares are trading, investors have no opinion on what the stock is really worth. It is as likely to have a high value as a low value as anything in between.[4] The Bayesian analysis uses this prior distribution to conclude that after investors observe the $12 price, the possibilities for fair value are narrowed to just $10/share and $15/share with equal probability.

Suppose now that Stock A is trading at $12/share and that Stock B is trading at $8/share. Because A and B have the same number of shares outstanding, their market capitalizations will be in the ratio 60/40. Investing 60 percent in A and 40 percent in B has four possible outcomes, as shown in **Table 2**, and the average return of the cap-weighted index is the same as that of the equally weighted index.

The analysis shows that capitalization weighting imposes no drag on expected return because capitalization weighting does not cause one to invest more in overvalued stocks and less in undervalued stocks. It invests the *same* proportions, here 60/40, without regard to undervaluation or overvaluation of the shares. Provided that A and B are randomly overvalued or undervalued by 20 percent, cap-weighted and equally weighted portfolios will have the same expected return regardless of the prices at which A and B are trading.

The Crux of the Issue

The crux of the issue is that the noisy market hypothesis effectively anchors on fair value—holding fair value fixed and using the probability distribution of the pricing error to deduce the probability distribution of market prices. To do so is to presuppose systematic reversals in stock prices, an assertion that does *not* follow from stocks being randomly mispriced. The big claim of the theory is that one can outperform cap-weighed indices *without knowing* fair value. If one does not know fair value, then even though prices may move toward fair value, the direction of that movement is random. To anchor on fair value is thus to contradict the going-in assumption of the noisy market hypothesis that we do not know fair value.

If all that one knows about a stock is its *current* price, the correct analysis is to hold that price fixed and use the probability distribution of the pricing error to deduce the probability distribution of the unknown fair value. As illustrated in the example of Companies A and B and established more formally in Appendix A, such an analysis shows that a company's market capitalization by itself does not predict the return on its shares. Because market capitalization does not reveal whether a stock is overvalued or undervalued, the random mispricing of stocks does not systematically shift the portfolio weights toward overvalued stocks.

Correlation of Pricing Error with Fair Value vs. with Market Value

Another way to state the preceding conclusion is in terms of the correlation of the pricing error with fair value and with market value. Fundamental indexing proponents argue that if a stock's pricing error is uncorrelated with its fair value, the pricing error must be correlated with its market value, which in turn, gives rise to capitalization weighting inducing a performance bias. This is not the case. The Bayesian analysis (with uninformed prior beliefs) shows that if the pricing error is uncorrelated with

Table 2. Cap-Weighted vs. Equally Weighted Returns: Bayesian Perspective

Outcome	Cap Weights	Probability	Return on Stock A	Return on Stock B	Cap-Weighted Return	Equally Weighted Return
Both overvalued	60/40	25%	–8.3%	–8.3%	–8.3%	–8.3%
Both undervalued	60/40	25	37.5	37.5	37.5	37.5
A overvalued, B undervalued	60/40	25	–8.3	37.5	10.0	14.6
A undervalued, B overvalued	60/40	25	37.5	–8.3	19.2	14.6
Expected return					14.6	14.6

the stock's fair value, the pricing error is also uncorrelated with the stock's market value, conditional on knowing market value. This lack of correlation between the pricing error and conditional market value is precisely why cap-weighted portfolio returns are not *a priori* biased downward.

The Noisy Market Hypothesis and the Time Series of Returns

A more elaborate version of the noisy market hypothesis relates to the time series of returns. The claim is that a share of stock that is randomly mispriced from period to period will have excess volatility, which implies mean reversion—or negative serial correlation—in its stock returns.

When asset returns are mean reverting over time, a rebalancing strategy will tend to outperform a buy-and-hold strategy (see Perold and Sharpe 1988). The reason is that a rebalancing strategy at the margin will buy assets that have underperformed and sell assets that have outperformed, trades that are good, on average, because of subsequent mean reversion in returns. Capitalization weighting is a buy-and-hold strategy and, in the presence of mean reversion, is thus likely to underperform a rebalancing strategy—before taking into account transaction costs.

I take issue with this argument on several grounds. First, stock returns being mean reverting does not create a case against capitalization weighting per se but against all buy-and-hold strategies *regardless of the initial proportions in which individual assets are held*.

Second, it is hard to see how mispricing errors can be random from one period to the next. Whatever is causing a stock to be overvalued today is likely to cause it to be overvalued tomorrow. Random but persistent misvaluation will cause mean reversion in returns over periods likely to be longer than a reasonable rebalancing interval.

Third, rather than causing mean reversion in returns, mispricing could go the other way and cause stocks to underreact to changes in fundamentals. In a world of underreacting stock prices (i.e., positive serial correlation/momentum of returns), buy-and-hold strategies will outperform rebalancing strategies. Rebalancing strategies will tend to be inferior in this case because they sell a stock after it has moved up, only to find that it continues to move up, on average, or they buy a stock after it has moved down, only to find that it continues to move down, on average.

And fourth, the empirical evidence on serial correlation of individual stock returns is, at best, inconclusive. Stocks tend to exhibit momentum effects (if at all) over monthly and annual periods, and they tend to exhibit mean-reversion effects (if at all) over longer intervals. Most importantly, if an investor knows something about the serial correlation of return of a particular stock, why wouldn't the investor exploit this knowledge directly? The appropriate strategy to exploit serial correlation may be a far cry from a simple rebalancing rule, particularly once transaction costs and taxes are taken into account.[5]

Fundamental Indexing As a Value Tilt

The supposition that capitalization weighting induces a mathematical headwind against performance is an important underpinning for the argument proponents are making for fundamental indexing. Additionally, fundamental indexers are proposing that if one is not going to invest according to capitalization weights, a good strategy is to tilt the portfolio toward "value stocks"—stocks with such characteristics as low P/Es and high dividend yields.[6] Clifford Asness (2006) and Jack Bogle and Burton Malkiel (2006) have explained eloquently how fundamental "indexing" is simply a particular packaging of quantitative value investing.

Historically, value stocks have generated higher-than-index returns, and the effect has been well documented and widely debated (see Fama and French 1992). At issue is whether value stocks have had high returns because they are riskier or because they are mispriced. If the effect is about risk, then fundamental indexers (and quantitative value investors generally) cannot expect to obtain high returns after adjusting for risk. If the effect is about mispricing, fundamental indexers will need to rely on a continuation of that *pattern* of mispricing in order to obtain high future returns—the pattern being that the market does not fully account for companies' book values, sales, earnings, and other readily obtainable fundamental information when determining stock prices. If value stocks are systematically mispriced, fundamental indexing may perform well—along with other value-oriented strategies—because it is exploiting this particular inefficiency, not because capitalization weighting, in and of itself, creates a performance bias.

Conclusion

Holding a stock in proportion to its capitalization weight does not change the likelihood that the stock is overvalued or undervalued. The notion that capitalization weighting imposes an intrinsic drag on performance is, accordingly, false. Fundamental indexing is a strategy of active security selection through investing in value stocks. It is a strategy not everyone can follow. Investors who have no skill in evaluating value tilts and other active strategies should hold the cap-weighted market portfolio.

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This article qualifies for 0.5 PD credit.

Appendix A. Formal Model

The formal model behind the example given in the main text is as follows.[7]

The equity market has N traded companies.

The price of the ith stock is P_i, and its fair value is P_i^*. The pricing error is

$$e_i = \frac{P_i}{P_i^*} - 1. \tag{A1}$$

Each company has one share outstanding. The share is infinitely divisible, so investors can buy any fraction of the company they wish. Because each company has one share outstanding, its market capitalization is simply its stock price, P_i. At current market prices, the capitalization weight of stock i is

$$W_i = \frac{P_i}{\sum_{j=1,N} P_j}. \tag{A2}$$

Each stock if fairly valued has the same known required return r. Moreover, stock i will trade for certain at price $P_i^*(1 + r)$ one period from now.

Investors observe the current price P_i but do not know P_i^*. Before observing P_i, investors have a prior distribution of P_i^* given by the probability density function, $f(P_i^*)$.

Conditional on P_i^*, the probability distribution of the pricing error is $g(e_i)$ and is unrelated to P_i^*. For the purpose at hand, it is not essential that the pricing error have expectation zero or be symmetrically distributed.

The probability densities $f(\cdot)$ and $g(\cdot)$ are common to all shares, but the pricing errors and fair values are independently distributed across shares.

The prior distribution $f(\cdot)$ is uninformed in that fair value P_i^* could be anything (over the range $P_i^* > 0$). Because stock prices tend to grow or shrink geometrically, it is sensible to assume that P_i^* is equally likely to lie in intervals that are geometrically evenly spaced—in other words, that P_i^* is as likely to lie between \$50 and \$100 as it is between \$100 and \$200, between \$200 and \$400, and so on. Therefore, $\log(P_i^*)$ is uniformly distributed between $\pm\infty$.

Let $h(P_i|P_i^*)$ denote the probability distribution of P_i conditional on knowing P_i^*. Thus,

$$h\left(P_i \middle| P_i^*\right) = \frac{g\left(P_i / P_i^* - 1\right)}{P_i^*}. \tag{A3}$$

Let $k(P_i^*|P_i)$ denote the probability distribution of P_i^* conditional on knowing P_i (the posterior distribution of P_i^*). By Bayes' Theorem, $k(P_i^*|P_i)$ is proportional to $h(P_i|P_i^*)f(P_i^*)$. With $\log(P_i^*)$ being uniformly distributed, $k(P_i^*|P_i)$ evaluates to $g(P_i/P_i^* - 1)P_i/(P_i^*)^2$.

Conditional on knowing P_i but not P_i^*, the return on stock i is $P_i^*(1 + r)/P_i - 1$. Integrating over P_i^* with respect to the density $k(P_i^*|P_i)$, and making a change of variable from P_i^* to e_i, shows that the expected conditional return on stock i, denoted m, can be expressed as

$$m = (1+r)E\left(\frac{1}{1+e_i}\right) - 1, \tag{A4}$$

where the expectation in this expression is taken with respect to the error density, $g(e_i)$.

A crucial point is that a stock's conditional expected return m is independent of its stock price P_i and, hence, of its market capitalization. Each stock has this same expected return m, and thus any portfolio—whether capitalization weighted or otherwise—will also have this expected return (14.6 percent in the example).

Therefore, even though individual stocks may have random pricing errors, market capitalization does not predict returns and capitalization weighting, in and of itself, does not create a performance drag.

Notes

1. The term "noisy market hypothesis" was coined by Jeremy Siegel in 2006.
2. "Fundamental Indexing" is a trademark of Research Affiliates, LLC.
3. The "rigorous" proofs Siegel is referring to are contained in Treynor (2005) and Hsu (2006).
4. As discussed formally in Appendix A, this analysis actually assumes a uniform prior on the log of fair value.
5. For an analysis of the optimal rebalancing strategy in the presence of long-term mean reversion in individual stock prices, see Jurek and Viceira (2006).
6. See, for example, Arnott, Hsu, and Moore (2005); Siegel (2006).
7. The basic setup here mirrors that in Hsu (2006).

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