An Illustrated Guide to the
ANALYTIC HIERARCHY PROCESS

Dr. Rainer Haas
Dr. Oliver Meixner
Institute of Marketing & Innovation
University of Natural Resources and Applied Life Sciences, Vienna

http://www.boku.ac.at/mi/

Do your decision conferences turn out like this?

WE WANT PROGRAM A!!

TOO BAD! WE WANT PROGRAM B!!

or does this happen?

COME ON IN THE WATER IS FINE!
DO YOUR RECOMMENDATIONS TURN OUT LIKE THIS?

BUT BOSS... THAT WAS MY BEST GUESS!

MAYBE YOU NEED A NEW APPROACH

GUESS AGAIN

... another way of decision making

I THINK I ‘LL TRY THE ANALYTIC HIERARCHY PROCESS (AHP) !!!
Dr. Thomas L. Saaty developed the process in the early 1970's and...

The process has been used to assist numerous corporate and government decision makers.

Some examples of decision problems:
- choosing a telecommunication system
- formulating a drug policy
- choosing a product marketing strategy
- ...
Let’s show how it works

PROBLEMS ARE DECOMPOSED INTO A HIERARCHY OF CRITERIA AND ALTERNATIVES

Criterion 1.1

Criterion 1
Criterion 2
... Criterion n

Problem

Criterion 1
Criterion 2
... Criterion n

Alternative 1
Alternative 2
... Alternative n

OKAY, HERE’S A DECISION PROBLEM WE FACE IN OUR PERSONAL LIVES
I SEE A NEW CAR IN YOUR FUTURE

STATE THE OBJECTIVE:
- SELECT A NEW CAR

DEFINE THE CRITERIA:
- STYLE, RELIABILITY, FUEL ECONOMY

PICK THE ALTERNATIVES:
- CIVIC COUPE, SATURN COUPE, FORD ESCORT, RENAULT CLIO

WHAT ABOUT COST?
(BE QUIET, WE'LL TALK ABOUT THAT LATER)
Select a new car

Style
- Civic
- Saturn
- Escort
- Clio

Reliability
- Civic
- Saturn
- Escort
- Clio

Fuel Economy
- Civic
- Saturn
- Escort
- Clio

THE INFORMATION IS THEN ARRANGED IN A HIERARCHICAL TREE

OBJECTIVE

CRITERIA

ALTERNATIVES

THIS INFORMATION IS THEN ARRANGED IN A HIERARCHICAL TREE

THE INFORMATION IS THEN SYNTHESIZED TO DETERMINE RELATIVE RANKINGS OF ALTERNATIVES

BOTH QUALITATIVE AND QUANTITATIVE CRITERIA CAN BE COMPARED USING INFORMED JUDGMENTS TO DERIVE WEIGHTS AND PRIORITIES
HOW DO YOU DETERMINE THE RELATIVE IMPORTANCE OF THE CRITERIA?

Here’s one way!

1. RELIABILITY IS 2 TIMES AS IMPORTANT AS STYLE
2. STYLE IS 3 TIMES AS IMPORTANT AS FUEL ECONOMY
3. RELIABILITY IS 4 TIMES AS IMPORTANT AS FUEL ECONOMY

HE’S NOT VERY CONSISTENT HERE ... THAT’S O.K.
PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED

1 equal  3 moderate   5 strong   7 very strong   9 extreme

<table>
<thead>
<tr>
<th></th>
<th>STYLE</th>
<th>RELIABILITY</th>
<th>FUEL ECONOMY</th>
</tr>
</thead>
<tbody>
<tr>
<td>STYLE</td>
<td>1/1</td>
<td>1/2</td>
<td>3/1</td>
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<tr>
<td>RELIABILITY</td>
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<td></td>
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<tr>
<td>FUEL ECONOMY</td>
<td>1/1</td>
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</table>
**Pairwise Comparisons**

*Using pairwise comparisons, the relative importance of one criterion over another can be expressed.*

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<td>FUEL ECONOMY</td>
<td>1/3</td>
<td>1/4</td>
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</table>

**How do you turn this matrix into ranking of criteria?**

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<td>1/3</td>
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</tbody>
</table>
AND THE SURVEY SAYS

EIGENVECTOR !!

ACTUALLY...

DR THOMAS L. SAATY, CURRENTLY WITH THE UNIVERSITY OF PITTSBURGH, DEMONSTRATED MATHEMATICALLY THAT THE EIGENVECTOR SOLUTION WAS THE BEST APPROACH.

REFERENCE: THE ANALYTIC HIERARCHY PROCESS, 1990, THOMAS L. SAATY

HERE’S HOW TO SOLVE FOR THE EIGENVECTOR:

1. A SHORT COMPUTATIONAL WAY TO OBTAIN THIS RANKING IS TO RAISE THE PAIRWISE MATRIX TO POWERS THAT ARE SUCCESSIVELY SQUARED EACH TIME.

2. THE ROW SUMS ARE THEN CALCULATED ANDNORMALIZED.

3. THE COMPUTER IS INSTRUCTED TO STOP WHEN THE DIFFERENCE BETWEEN THESE SUMS IN TWO CONSECUTIVE CALCULATIONS IS SMALLER THAN A PRESCRIBED VALUE.

SHOW ME AN EXAMPLE

SAY WHAT!
IT’S MATRIX ALGEBRA TIME !!!

<table>
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<td>3/1</td>
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<tr>
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<td>2/1</td>
<td>1/1</td>
<td>4/1</td>
</tr>
<tr>
<td>FUEL ECONOMY</td>
<td>1/3</td>
<td>1/4</td>
<td>1/1</td>
</tr>
</tbody>
</table>

For now, let’s remove the names and convert the fractions to decimals:

\[
\begin{bmatrix}
1.0000 & 0.5000 & 3.0000 \\
2.0000 & 1.0000 & 4.0000 \\
0.3333 & 0.2500 & 1.0000
\end{bmatrix}
\]

Step 1: Squaring the matrix

\[
\begin{bmatrix}
1.0000 & 0.5000 & 3.0000 \\
2.0000 & 1.0000 & 4.0000 \\
0.3333 & 0.2500 & 1.0000
\end{bmatrix}
\]

This times\[\begin{bmatrix}
1.0000 & 0.5000 & 3.0000 \\
2.0000 & 1.0000 & 4.0000 \\
0.3333 & 0.2500 & 1.0000
\end{bmatrix}\]

This results in this

\[
\begin{bmatrix}
3.0000 & 1.7500 & 8.0000 \\
5.3332 & 3.0000 & 14.0000 \\
1.1666 & 0.6667 & 3.0000
\end{bmatrix}
\]

I.E. (1.0000 * 1.0000) + (0.5000 * 2.0000) + (3.0000 * 0.3333) = 3.0000
STEP 2: NOW, LET’S COMPUTE OUR FIRST EIGENVECTOR (TO FOUR DECIMAL PLACES)
FIRST, WE SUM THE ROWS

\[
\begin{align*}
3.0000 & + 1.7500 & + & 8.0000 & = 12.7500 & 0.3194 \\
5.3332 & + 3.0000 & + & 14.0000 & = 22.3332 & 0.5595 \\
1.1666 & + 0.6667 & + & 3.0000 & = 4.8333 & 0.1211
\end{align*}
\]
SECOND, WE SUM THE ROW TOTALS

\[
\begin{array}{c}
39.9165 \\
1.0000
\end{array}
\]
FINALLY, WE NORMALIZE BY DIVIDING THE ROW SUM BY THE ROW TOTALS (I.E. 12.7500 DIVIDED BY 39.9165 EQUALS 0.3194)

\[
\begin{array}{c}
0.3194 \\
0.5595 \\
0.1211
\end{array}
\]

THE RESULT IS OUR EIGENVECTOR (A LATER SLIDE WILL EXPLAIN THE MEANING IN TERMS OF OUR EXAMPLE)

This process must be iterated until the eigenvector solution does not change from the previous iteration (remember to four decimal places in our example)

CONTINUING OUR EXAMPLE, AGAIN, STEP 1: WE SQUARE THIS MATRIX

\[
\begin{align*}
3.0000 & & 1.7500 & & 8.0000 \\
5.3332 & & 3.0000 & & 14.0000 \\
1.1666 & & 0.6667 & & 3.0000
\end{align*}
\]
WITH THIS RESULT

\[
\begin{align*}
27.6653 & & 15.8330 & & 72.4984 \\
48.3311 & & 27.6662 & & 126.6642 \\
10.5547 & & 6.0414 & & 27.6653
\end{align*}
\]
Again Step 2: Compute the eigenvector (to four decimal places)

\[
\begin{array}{ccc}
27.6653 & + & 15.8330 & + & 72.4984 & = & 115.9967 & \pm & 0.3196 \\
48.3311 & + & 27.6662 & + & 126.6642 & = & 202.6615 & \pm & 0.5584 \\
10.5547 & + & 6.0414 & + & 27.6653 & = & 44.2614 & \pm & 0.1220 \\
\hline
\end{array}
\]

TOTALS 362.9196 \pm 1.0000

Compute the difference of the previous computed eigenvector to this one:

\[
\begin{array}{ccc}
0.3194 & - & 0.3196 & = & -0.0002 \\
0.5595 & - & 0.5584 & = & 0.0011 \\
0.1211 & - & 0.1220 & = & -0.0009 \\
\end{array}
\]

To four decimal places there's not much difference. How about one more iteration?

I surrender!!
Don't make me compute another eigenvector.

Okay, okay. Actually, one more iteration would show no difference to four decimal places.

Let's now look at the meaning of the eigenvector.
HERE’S OUR PAIRWISE MATRIX WITH THE NAMES

<table>
<thead>
<tr>
<th></th>
<th>STYLE</th>
<th>RELIABILITY</th>
<th>FUEL ECONOMY</th>
</tr>
</thead>
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<tr>
<td>STYLE</td>
<td>1/1</td>
<td>1/2</td>
<td>3/1</td>
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<td>RELIABILITY</td>
<td>2/1</td>
<td>1/1</td>
<td>4/1</td>
</tr>
<tr>
<td>FUEL ECONOMY</td>
<td>1/3</td>
<td>1/4</td>
<td>1/1</td>
</tr>
</tbody>
</table>

AND THE COMPUTED EIGENVECTOR GIVES US THE RELATIVE RANKING OF OUR CRITERIA

- STYLE: 0.3196 ➔ THE SECOND MOST IMPORTANT CRITERION
- RELIABILITY: 0.5584 ➔ THE MOST IMPORTANT CRITERION
- FUEL ECONOMY: 0.1220 ➔ THE LEAST IMPORTANT CRITERION

NOW BACK TO THE HIERARCHICAL TREE...

HERE’S THE TREE WITH THE CRITERIA WEIGHTS

OBJECTIVE

Select a new car
1.00

CRITERIA

- Style: 0.3196
  - Civic
  - Saturn
  - Escort
  - Clio

- Reliability: 0.5584
  - Civic
  - Saturn
  - Escort
  - Clio

- Fuel Economy: 0.1220
  - Civic
  - Saturn
  - Escort
  - Clio

WHAT ABOUT THE ALTERNATIVES?

I’M GLAD YOU ASKED...
IN TERMS OF STYLE, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER

<table>
<thead>
<tr>
<th></th>
<th>CIVIC</th>
<th>SATURN</th>
<th>ESCORT</th>
<th>CLIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIVIC</td>
<td>1/1</td>
<td>1/4</td>
<td>4/1</td>
<td>1/6</td>
</tr>
<tr>
<td>SATURN</td>
<td>4/1</td>
<td>1/1</td>
<td>4/1</td>
<td>1/4</td>
</tr>
<tr>
<td>ESCORT</td>
<td>1/4</td>
<td>1/4</td>
<td>1/1</td>
<td>1/5</td>
</tr>
<tr>
<td>CLIO</td>
<td>6/1</td>
<td>4/1</td>
<td>5/1</td>
<td>1/1</td>
</tr>
</tbody>
</table>

AND...

IN TERMS OF RELIABILITY, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER

<table>
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<th>ESCORT</th>
<th>CLIO</th>
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<tbody>
<tr>
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<td>1/1</td>
<td>2/1</td>
<td>5/1</td>
<td>1/1</td>
</tr>
<tr>
<td>SATURN</td>
<td>1/2</td>
<td>1/1</td>
<td>3/1</td>
<td>2/1</td>
</tr>
<tr>
<td>ESCORT</td>
<td>1/5</td>
<td>1/3</td>
<td>1/1</td>
<td>1/4</td>
</tr>
<tr>
<td>CLIO</td>
<td>1/1</td>
<td>1/2</td>
<td>4/1</td>
<td>1/1</td>
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</table>

ITS MATRIX ALGEBRA TIME!!!
Computing the eigenvector determines the relative ranking of alternatives under each criterion.

<table>
<thead>
<tr>
<th>RANKING</th>
<th>STYLE</th>
<th>RANKING</th>
<th>RELIABILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>CIVIC</td>
<td>.1160</td>
<td>1 CIVIC</td>
</tr>
<tr>
<td>2</td>
<td>SATURN</td>
<td>.2470</td>
<td>2 SATURN</td>
</tr>
<tr>
<td>4</td>
<td>ESCORT</td>
<td>.0600</td>
<td>4 ESCORT</td>
</tr>
<tr>
<td>1</td>
<td>CLIO</td>
<td>.5770</td>
<td>3 CLIO</td>
</tr>
</tbody>
</table>

What about fuel economy? Another good question...

As stated earlier, AHP can combine both qualitative and quantitative information.

Fuel economy information is obtained for each alternative:

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Fuel Economy (Miles/Gallon)</th>
<th>Normalized</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIVIC</td>
<td>34</td>
<td>0.3010</td>
</tr>
<tr>
<td>SATURN</td>
<td>27</td>
<td>0.2390</td>
</tr>
<tr>
<td>ESCORT</td>
<td>24</td>
<td>0.2120</td>
</tr>
<tr>
<td>CLIO</td>
<td>28</td>
<td>0.2480</td>
</tr>
<tr>
<td></td>
<td>113</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Normalizing the fuel economy info allows us to use it with other rankings.
Select a new car

1.00

CRITERIA

HERE’S THE TREE
WITH ALL THE WEIGHTS

ALTERNATIVES

OKAY, NOW WHAT? I THINK WE’RE READY FOR THE ANSWER...

A LITTLE MORE MATRIX ALGEBRA GIVES US THE SOLUTION:

I.E. FOR THE CIVIC (.1160 * .3196) + (.3790 * .5584) + (.3010 * .1220) = .3060

AND THE WINNER IS !!!

THE CLIO IS THE HIGHEST RANKED CAR
IN SUMMARY, THE ANALYTIC HIERARCHY PROCESS PROVIDES A LOGICAL FRAMEWORK TO DETERMINE THE BENEFITS OF EACH ALTERNATIVE

1. Clio .3280
2. Civic .3060
3. Saturn .2720
4. Escort .0940

WHAT ABOUT COSTS?

SKEPTIC-GATOR

WELL, I’LL TELL YOU...

ALTHOUGH COSTS COULD HAVE BEEN INCLUDED, IN MANY COMPLEX DECISIONS, COSTS SHOULD BE SET ASIDE UNTIL THE BENEFITS OF THE ALTERNATIVES ARE EVALUATED

OTHERWISE THIS COULD HAPPEN...

YOUR PROGRAM COST TOO MUCH I DON’T CARE ABOUT ITS BENEFITS

DISCUSSING COSTS TOGETHER WITH BENEFITS CAN SOMETIMES BRING FORTH MANY POLITICAL AND EMOTIONAL RESPONSES
WAYS TO HANDLE BENEFITS AND COSTS INCLUDE THE FOLLOWING:

1. GRAPHING BENEFITS AND COSTS OF EACH ALTERNATIVE

2. BENEFIT TO COST RATIOS

3. LINEAR PROGRAMMING

4. SEPARATE BENEFIT AND COST HIERARCHICAL TREES AND THEN COMBINE THE RESULTS

IN OUR EXAMPLE...

LET’S USE BENEFIT TO COST RATIOS

<table>
<thead>
<tr>
<th></th>
<th>COST $</th>
<th>NORMALIZED COSTS</th>
<th>BENEFIT - COST RATIOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. CLIO</td>
<td>18,000</td>
<td>.3333</td>
<td>.3280 / .3333 = .9840</td>
</tr>
<tr>
<td>2. CIVIC</td>
<td>12,000</td>
<td>.2222</td>
<td>.3060 / .2222 = 1.3771</td>
</tr>
<tr>
<td>3. SATURN</td>
<td>15,000</td>
<td>.2778</td>
<td>.2720 / .2778 = .9791</td>
</tr>
<tr>
<td>4. ESCORT</td>
<td>9,000</td>
<td>.1667</td>
<td>.0940 / .1667 = .5639</td>
</tr>
<tr>
<td></td>
<td>54,000</td>
<td>1.0000</td>
<td></td>
</tr>
</tbody>
</table>

(RENEMEMBER THE BENEFITS WERE DERIVED EARLIER FROM THE AHP)

AND...

THE CIVIC IS THE WINNER WITH THE HIGHEST BENEFIT TO COST RATIO
AHP CAN BE USED FOR VERY COMPLEX DECISIONS

MANY LEVELS OF CRITERIA AND SUBCRITERIA CAN BE INCLUDED

GOAL

HERE’S SOME EXAMPLES

AHP CAN BE USED FOR A WIDE VARIETY OF APPLICATIONS

STRATEGIC PLANNING
RESOURCE ALLOCATION
SOURCE SELECTION
BUSINESS/PUBLIC POLICY
PROGRAM SELECTION
AND MUCH MUCH MORE...